1. **Design Specification for plate girder bridge**

The basic difference between beam and girder is their web slenderness ratio $h/t_w$. When this value is greater than $760/\sqrt{F_b}$, the beam is normally called girder. When no local buckling is allowed to form on the compression flange, the girder is called compacted section otherwise it will be called as non-compacted section. Depending on the type of girder, use Chapter B, F and Chapter G accordingly.

2. **Design method**

3. **Work out Examples**

Design a Welded plate girder with a simply support span of 56 ft to support a uniform load of 3kips/ft (including girder self weight) and two concentrated loads of 75 kips located 20 ft from each end. The compression flange is laterally supported only at point of concentrated load. Use ASTM A 36 steel and the AISC design guideline. Use also AASHTO bridge design manual whenever deemed necessary.

**Solution:**
$F_y = 36$ ksi \hspace{2cm} [From Table-1]

Max shear = 159 kips

Max Moment = 2676 kip-ft

### A. Preliminary web design:

1. Select a girder with a depth of about $\frac{1}{8}$ of the span:
   
   \[ L = 56 \text{ ft} \]
   
   \[ L = \frac{56 \times 12}{8} = 84" \]

   Take a trial depth of 80"

2. For no reduction in allowable bending stress, $F_b$ in flange:
   
   \[ \frac{h}{t_w} \leq \frac{760}{\sqrt{F_b}} = \frac{760}{\sqrt{22}} = 162 \quad [F_b = 0.6F_y, \quad \text{Eq. F1-5}] \]
   
   \[ \therefore t_w = 0.494" \]

   The corresponding thickness of the web = 0.494"

3. Maximum ratio of clear distance between flange to web thickness:
   
   \[ \frac{h}{t} \leq \frac{14000}{F_y (F_y + 16.5)} = 322 \quad [\text{Chap-G, Eq. G1-1}] \]

   The corresponding minimum web thickness, $t=0.248"$

   Try a web plate $\frac{1}{4} \times 80$: $A_w = 0.25 \times 80 = 20$ in$^2$

   \[ \frac{h}{t} = \frac{80}{\frac{1}{4}} = 320 > 162 \]

   Allowable flange stress criteria will be reduced (see B-3).

### B. Preliminary flange design:

1. Required flange area:

   An approximate formula for the area of one flange is:

   \[ A_f = \frac{M}{F_y h} = \frac{2676 \times 12}{22 \times 80} = 18.24 \text{ in}^2 \]

   $b_f = 20"$  
   $t_f = \frac{1}{8}h"$  
   $t_w = \frac{1}{4}h"$  
   $h = 80"$  
   $t_f = \frac{1}{8}h"$  
   $b_f = 20"$
Try a plate with $\frac{7}{8} \times 20 = 17.5$ in$^2$

Section properties:

i) Moment of Inertia:

$$I_x = \left[ \frac{80^3}{4 \times 12} + 2 \times 20 \times 40.438^2 \right]$$

[Neglect the $I_x$ for flanges]

$$= 76,900 \text{ in}^4$$

ii) Section Modulus:

$$S_x = \frac{I_x}{c} = \frac{67,900}{40.875} = 1661 \text{ in}^3$$

Moment of Inertia of flange plus $\frac{1}{6}$ of web about y axis (weak axis)

$$I_y = \frac{7}{8} \times \frac{20^3}{12} = 583 \text{ in}^4$$

Area = $A_y + A_w = 17.5 + \frac{80}{6} \times \frac{1}{4} = 20.83 \text{ in}^2$

Radius of gyration, $r_y = \frac{I_y}{A} = \sqrt{\frac{835}{20.83}} = 5.3 \text{ in}$

2. **Check the $\frac{1}{2}$ flange width/thickness ratio**

The permissible $\frac{1}{2}$ flange width/thickness ratio is:

$$\frac{b_f}{2t_f} < \frac{95}{\sqrt{F_y}}$$

$$\frac{95}{\sqrt{F_y}} = 15.8$$

[Commentary C-F2]

$$\frac{b_f}{2t_f} = 11.4 < 15.8 \text{ OK}$$

3. **Check the allowable bending stress in flange:**

i) For 16 ft middle panel:

Maximum bending stress at mid-span:

$$f_b = \frac{M_{\text{max}}}{S_x} = \frac{2676 \times 12}{1661} = 19.3 \text{ ksi}$$

The moment between the ends is greater than the moments at the ends, $M_1$ and $M_2$

$$i.e \, M_{\text{max}} > M_1 \text{ and } M_2 \therefore C_b = 1$$

$$\sqrt{\frac{102 \times 10^2 C_b}{F_y}} = 53 \sqrt{C_b}$$

$$\frac{l}{r_y} = \frac{16 \times 12}{5.3} = 36.2 < 53 \text{ OK}$$

$$\therefore \text{The allowable stress based upon lateral buckling criteria:}$$

$$F_y = 0.6F_b = 21.6 \text{ ksi}$$

Reduced allowable bending stress in compression flange:

$$F_y' \leq F_y \times R_{pf} \times R_v$$

Where,
Design of Plate Girder Bridge by AISC ASD method

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\[
R_{pg} = 1 - 0.0005 \frac{A_t}{A_f} \left( \frac{h}{t} \sqrt{F_y} \right) \leq 1.0 \quad \text{[Eq. G2-1]}
\]

\[ R_s = 1 \quad \text{(for non-hybrid girder section)} \]

\[
F_s^* = F_b \left[ 1 - 0.0005 \frac{20}{17.5} \left( 320 - \frac{760}{\sqrt{21.6}} \right) \right]
\]

\[ = 19.67 \text{ ksi} > 19.3 \text{ OK} \]

ii) For 20 ft end panels:

Maximum bending stress at end-span:

\[
f_e = \frac{M}{S} = \frac{2580 \times 12}{1661} = 18.6 \text{ ksi}
\]

\[
C_b = 1.75 + 1.05 \frac{M_1}{M_2} + 0.3 \left( \frac{M_1}{M_2} \right)^2
\]

Where \( M_1 = 0 \); then \( \frac{M_1}{M_2} = 0 \) \quad \therefore \quad C_b = 1.75

\[
\sqrt{\frac{102 \times 10^4 C_b}{F_y}} = 53\sqrt{1.75} = 70.41
\]

\[
\frac{l}{r_b} = \frac{20 \times 12}{5.3} = 45.3 < 70.41 \text{ OK}
\]

\[ F_s^* = 19.67 \]

\[ : f_b < F_s^* \text{ OK} \]

Use:

- for Web: one plate \( \frac{1}{4} \times 80 \)
- for Flange: two plate \( 7/8 \times 20 \)

C. Stiffener requirements:

1. Bearing Stiffener:

   i) End bearing stiffeners are required at unframed girder ends

   ii) Check bearing under concentrated load:

   Assume point bearing and \( \frac{1}{4} \) in. web-to-flange welds.

   Local web yielding:

   \[
   \frac{R}{t_w (N + 5k)} \leq 0.66 F_y \quad \text{[Eq. K1-2]}
   \]

   Where,

   \[ R = \text{concentrated load or reaction, kip} \]
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$t_w =$ thickness of the web, inch

$N =$ length of bearing (not less than $k$ for end reactions), inch

$k =$ distance from outer face of flange to web toe of fillet, inch

\[
\frac{R}{t_w(N + 5k)} = \frac{75}{\frac{1}{4}(0 + 5 \times 1)} = 60 > 0.66 \times 36 = 23.8 \text{ ksi} \quad \text{Not Governed}
\]

[Note: If local web yielding criterion is satisfied, criteria for crippling in Sec K1.4 and Sec K1.5 would have to be checked]

\[
\therefore \text{ Provide bearing stiffener under concentrated load}
\]

2. Intermediate Stiffeners:

Stiffeners are not required if,

\[
\frac{h}{t_w} < 260 \quad \text{and} \quad f_v < F_v,
\]

where, $f_v =$ Actual shear stress

$F_v =$ Allowable shear stress

Here, \[
\frac{h}{t_w} = \frac{80}{\frac{1}{4}} = 320 > 260 \quad \text{So, stiffeners are required.}
\]

\[
f_v = \frac{V}{A_v} = \frac{159}{20} = 7.95 \text{ ksi}
\]

Find the allowable shear stress,

\[
F_v = \frac{F_s}{2.89} C_v \leq 0.40 F_s
\]

Where,

\[
C_v = \frac{45,000k_e}{F_s \left( \frac{\%}{\%} \right)^2} \quad \text{when} \quad \% < 0.8 \quad \text{and}, \quad k_e = 4.00 + \frac{5.34}{\left( \% \right)^2} \quad \text{when} \quad \% \leq 1.0
\]

\[
C_v = \frac{190k_e}{F_s \left( \frac{\%}{\%} \right)^2} \quad \text{when} \quad \% > 0.8 \quad \text{and}, \quad k_e = 5.35 + \frac{4.00}{\left( \% \right)^2} \quad \text{when} \quad \% > 1.0
\]

$t_w =$ thickness of the web, inch

$a =$ clear distance between transverse stiffeners, inch

$h =$ clear distance between flanges at the section under investigation, inch

Try, $a = 30$ inch

\[
\frac{a}{h} = \frac{30}{80} = 0.375
\]

\[
k_e = 4.00 + \frac{5.34}{\left( \% \right)^2} = 41.97
\]

\[
C_v = \frac{45,000k_e}{F_s \left( \frac{\%}{\%} \right)^2} = \frac{45,000 \times 41.97}{36 \times 320^2} = 0.512 < 0.8 \quad \text{OK}
\]

\[
F_v = \frac{F_s}{2.89} C_v = \frac{36}{2.89} 0.512 = 6.38 \text{ ksi}
\]

\[
\leq 0.40 F_s = 14.4 \text{ ksi} \quad \text{OK}
\]

Shear stress at a distance 30 in from the end,

\[
f_v = \frac{V}{A_v} = \frac{159 - 2.5 \times 3}{20} = 7.575 \text{ ksi} \quad \text{Not satisfied. Design should be revised.}
\]
Try, \( a = 25 \) inch

\[
\frac{a}{h} = \frac{25}{80} = 0.3125
\]

\( k_v = 4.00 + \frac{5.34}{(0.3125)} = 58.68 \)

\[
C_v = \frac{45,000k_v}{F_v(\%)} = \frac{45,000 \times 58.68}{36 \times 320^2} = 0.716 < 0.8 \text{ OK}
\]

\[
F_v = \frac{F_v}{2.89} \quad C_v = \frac{36}{2.89} \quad 0.716 = 8.92 \text{ ksi} \leq 0.40F_v = 14.4 \text{ ksi OK}
\]

Shear stress at a distance 25 in from the end,

\[
f_v = \frac{V}{A_v} = \frac{159 - \frac{23}{36} \times 3}{20} = 7.64 \text{ ksi} < F_v \text{ OK}
\]

3. Locate the position of remaining intermediate stiffeners:

\[
\frac{a}{h} \leq \left[ \frac{260}{\left(\frac{h/t_v}{320}\right)} \right]^2 = \left(\frac{260}{320}\right)^2 = 0.66 \; \text{; } a = \text{ spacing of the intermediate stiffener [Eq.F5-1]}
\]

\[
\therefore a \leq 0.66 \times 80 = 52.8 \text{ inch}
\]

i) For spacing between the first intermediate stiffener and the concentrated load:

For \( \frac{a}{h} = 0.66 \) and \( \frac{h}{t_v} = 320 \), \( F_v = 9.7 \) ksi \[Table 2-36, Page 2-234\]

[Note: if the value cannot be obtained directly from the table, use interpolation]

Alternatively you can use the following formula;

\[
F_v = \frac{F_v}{2.89} \left[ C_v + \frac{1 - C_v}{1.15 \sqrt{1 + (\%)}^2} \right] \leq 0.40F_v \quad \text{[Eq.G3-1]}
\]

Using 5 stiffeners at a spacing of 43 inch will furnish the remaining intermediate stiffeners (as shown in Fig. below).

\[ a = \frac{20 \times 12 - 25}{5} = 43 \text{ inch} \]
For Spacing in the mid 16 ft panel:

Note: here bending stress is critical, so check for bending stress. Use Eq. for tension filed action.

Here also, \( a \leq 52.8'' \)

Use 3 stiffeners at a distance of 48” c/c.

[If 2 stiffeners are used, the spacing will be 64” which is greater than 52.8”]

Check interaction at concentrated load in tension field panel:

Allowable bending tensile stress in the web:

\[
F_{bh} = \left( 0.825 - 0.375 \frac{F_v}{F_y} \right) F_y \leq 0.60F_y \quad \text{[Eq. G5-1]}
\]

Here,

\( F_v \) = computed average web shear stress (total shear divided by web area) ksi

\( F_y \) = allowable web shear stress according to Eq. G3-1, ksi

\[
F_v = \frac{F_y}{2.89} \left[ C_v + \frac{1-C_v}{1.15\sqrt{1+\left(\frac{\%}{\%}\right)^2}} \right] \leq 0.40F_y
\]

\( a = 48'' \) : \( \% = \frac{48}{a} = 0.6 \)

\( k_v = 4.00 + \frac{5.34}{(\%)} = 18.83 \)

\( C_v = \frac{45,000k}{F_y(\%)} = \frac{45,000 \times 18.83}{36 \times 320^2} = 0.23 < 0.8 \ \text{OK} \)

\[
F_i = \frac{36}{2.89} \left[ 0.23 + \frac{1-0.23}{1.15\sqrt{1+(0.6)^2}} \right] = 10 \ \text{ksi}
\]

\[
F_{bh} = \left( 0.825 - 0.375 \frac{4.95}{10.0} \right) F_y = 0.64F_y > 0.60F_y
\]

\( \therefore F_{bh} = 0.6F_y = 21.6 \approx 22 \)

Actual bending stress in the web \( f_b \) should not be greater than \( F_{bh} \).

\[
f_v = \frac{My}{I} = \frac{2676 \times 12 \times 40}{67900} = 18.91 < 22 \ \text{ksi OK}
\]

So, the above spacing for the intermediate stiffeners are acceptable.

4. Select the size of the intermediate stiffener:

i) Stiffener area:

\[
A_u = \frac{1-C_v}{2} \left[ \frac{a}{h} - \left(\frac{\%}{\%}\right)^2 \right] YDht
\]
Where,

\[ Y = \text{ratio of yield stress of web steel to yield stress of stiffener steel} \]

\[ D = 1.0 \text{ for stiffener furnished in pair} \]

\[ = 1.8 \text{ for single angle stiffener} \]

\[ = 2.4 \text{ for single plate stiffener} \]

Stiffener steel area can be determined as % of web steel from table 2-36. [Page 2-234]

For \( Y_h = (48+43)/(2\times80) = 0.57 \) and \( Y_i = 320, A_{st} = 10.7\% \)

\[ \therefore A_{st} = 0.107 \times ht = 0.107 \times 80 \times \frac{1}{4} = 2.14 \text{ in}^2 \]

When the shear stress in a panel is less than the allowable shear stress, the reduction of gross area is permitted in proportion.

So, the actual area required, \( A_{st} = 2.14 \frac{F}{D} = \frac{2.14 \times 7.64}{10.0} \times 1 = 1.63 \text{ in}^2 \)

Try with two plate \( \frac{1}{4} \times 4 = 2 \text{ in}^2 > 1.63 \text{ in}^2 \) OK

**Check:**

ii) Width thickness ratio = \( b/t = 4/\frac{1}{4} = 16 \approx 95 \text{ OK} \) [Table B5.1, Page 5-36]

iii) Moment of Inertia, \( I_{st} \geq \left( \frac{Y_{st}}{Y_f} \right)^4 = (\frac{95}{12})^4 = 6.55 \text{ in}^4 \)

\( I_{st} \text{ furnished} = \frac{1}{12} \times 0.25 \left( 2 \times 4 + 0.25 \right)^3 = 11.7 \text{ in}^4 > 6.55 \text{ OK} \)

iv) Required length, \( L_{reqd} = h - 4t = 80 - 4 \times \frac{1}{4} = 79'' \), Furnish a length of 6'- 7”

Use for intermediate stiffeners: Two plates having dimensions of \( \frac{1}{4} \times 4 \times 6'-7'' \)

Use fillet weld for fixing the stiffener with compression flange and web.

5. **Design bearing stiffener:**

Since bearing stiffener must extend approximately to edges of the flange plates, Try with two \( \frac{1}{2} \times 8 \) plates

i) Check for width/thickness ratio: \( b/t = 8/\frac{1}{4} = 16 \approx 15.8 \text{ OK} \)

ii) Check the web crippling:

1) For UDL of 3 kips/ft load:

\[ f_a \leq \left[ 5.5 + \frac{4}{(\%)} \right] \left[ \frac{10,000}{(\%)} \right] \]

\[ f_a = \frac{P}{bt} = \frac{3}{12 \times \frac{1}{4}} = 1.0 \text{ ksi} \]

Again,
\[
\left[ 5.5 + \frac{4}{(\%)} \right] 10,000 \left( \frac{\%}{(\%)} \right)^2 = \left[ 5.5 + \frac{4}{(\%)} \right] 10,000 \left( \frac{\%}{(\%)} \right)^2 = 1.88 \text{ ksi} > 1.0 \text{ OK}
\]

2) For concentrated load of 159 kips at the ends:

\[
I = \frac{1}{12} \times 0.5 \times 16.25^3 = 179 \text{ in}^4
\]

\[
A_{\text{eff}} = 2 \times 0.5 \times 8 + 12 \times 0.25 \times 0.25 = 8.75 \text{ in}^2
\]

\[
r = \sqrt{\frac{I}{A_{\text{eff}}}} = \frac{179}{8.75} = 4.52 \text{ in} \quad \text{[K1.8]}
\]

\[
\frac{Kl}{r} = \frac{\frac{3}{4} \times 80}{4.52} = 13.3
\]

Allowable stress: \( F_a = 21.0 \text{ ksi} \) \quad \text{[Table C-36]}

Actual stress: \( f_a = \frac{P}{A_{\text{eff}}} = \frac{159}{8.75} = 18.2 \text{ ksi} < F_a \) \text{ OK}

Use two plates of \( \frac{1}{2} \times 8 \times 8'' \) as end bearing stiffeners with closely fit against the flanges that receive the concentrated loads. Use the same size of bearing stiffener under concentrated load, 20 ft. from the ends.

**APPENDIX**

1. Tables
2. Chapter B, F, G
3. Commentary B, F, G