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Influence of Stress Singularity on Transversely Isotropic Piezo-ceramic Bonded Joints

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Abstract

In recent years, intelligent or smart structures and systems have become an emerging new research area. Piezo-ceramics material, due to its characteristic direct-converse piezoelectric effect, has naturally received considerable attentions. The stress concentrations caused by mechanical or electric loads may lead to crack initiation and extension, and sometimes the stress concentrations may be high enough to fracture the material parts. Stress singularity of bonded joints is one of the main factors responsible for debonding under loading. However, the stress singularity for transversely isotropic piezo-ceramic bonded joints has not been made clear until now. In this paper, order of singularity and intensity of singularity in piezo-ceramic bonded joints is analyzed. Eigen analysis based on finite element method is used for order of singularity analysis of the joints. The distributions of stress and electric displacement in the singular field for the piezo-ceramic bonded structure are obtained by using the boundary element method. It is shown from the numerical result that the stress and electric displacement increase with the thickness of upper material and have larger value near the vertex of the interface. Therefore, there is a highest possibility to debond and delamination occurs at the corner of thick transversely isotropic piezo-ceramic bonded joints.

Keywords: *Bonded joint, Boundary element method, Eigenanalysis, Piezo-ceramic, Stress singularity.*

I. INTRODUCTION

The stress singularity fields are one of the main factors responsible for debonding under mechanical or thermal loading. Stress singularity frequently occurs at a vertex in an interface of joints due to a discontinuity of materials. Piezo-ceramic materials are playing a key role as active components in many fields of engineering and technology such as electronics, laser, microwave infrared, navigation and biology [1]. The stress distribution near the vertex in the interface of joints is very important to maintain the reliability of joints. Mechanical stress occurs in piezo-ceramic material for any electric input.

The stress concentrations caused by mechanical or electric loads may lead to crack initiation and extension, and sometimes the stress concentrations may be high enough to debond the material parts. Reliable service lifetime predictions of piezo-ceramic components demand a complete understanding of the debonding processes of these materials. When two materials are joined, a free-edge stress singularity usually develops at the intersection of the interface and the free surface.

The stress distributions around the vertex were determined using a boundary element method (BEM).

Koguchi [2] determined the intensity of singularity by fitting the stress profile obtained from BEM analysis with a least squares method. Constable [3] proposed a method to compute their singularity exponents and the associated angular singular functions. Their method was particularly useful with anisotropic materials and allowed to follow the dependency of singularity exponents along a curved edge.

Wang and Chen [4] and Wang and Zheng [5] studied the solution for a point force applied on the boundary of a transversely isotropic piezoelectric half-space and derived the general solution of the equations of equilibrium. Ding [6] systematically studied the general solutions of equations for transversely isotropic piezoelectric materials and obtained the solutions of a half-space. Ding [7] obtained the solutions for transversely isotropic piezoelectric media by using body potential theory and constructing a kind of harmonic function. Lee and Jiang [8] obtained the boundary integral equation and two-dimensional solution by using the double Fourier transform, by considering one case of the eigenvalues.

Recently, Hwu [9] and Ikeda [10] proposed the solution of singular stress field and its SIFs of an interfacial corner of a 2D dissimilar piezoelectric material joint using extended Stroh formalism.

At present, no clear picture exists of the problem of intensity of singularity for transversely isotropic piezo-ceramic bonded joints. Therefore, the influence of intensity of singularity at a vertex in transversely isotropic piezo-ceramics bonded joint is analyzed in this study.

II. FORMULA OF ANALYSIS

A. The Basic Formula

In the absence of body forces and free charges, the governing equations of three-dimensional piezoelectric materials are expressed as follows:

$$\sigma_{ij,j} = 0, \qquad D_{i,i} = 0 \tag{1}$$

where, σ_{ij} and D_i are stress tensor and electric displacement vector, respectively. These equations are the elastic equilibrium equations and Gauss's law of electrostatics, respectively. The constitutive relations are expressed as follows:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \qquad D_i = e_{ikl}\varepsilon_{kl} - \chi_{ik}E_k \tag{2}$$

whereε_{kl} is strain tensor, which is the mechanical field variables, E_k is the electric field, c_{ijkl} is the elastic constant, e_{kij}(e_{ikl}) and χ_{ik} are the piezoelectric constant and electric permittivity (dielectric constant), respectively. The elastic strain-displacement and electric field-potential equations are expressed as follows:

$$\varepsilon_{ij} = \tfrac{1}{2}\left(u_{j,i} + u_{i,j}\right), \qquad E_i = -\phi_{,i} \tag{3}$$

wherein and ϕ are the elastic displacement and electric potential, respectively.

According to Ding [6], the fundamental solutions of the governing differential equations for the transversely isotropic piezoelectric material are shown as follows:

$$u_x = \sum_{i=1}^{3} \frac{\partial \psi_i}{\partial x} - \frac{\partial \psi_o}{\partial y}$$

$$u_y = \sum_{i=1}^{3} \frac{\partial \psi_i}{\partial y} + \frac{\partial \psi_o}{\partial x} \qquad (s_1 \neq s_2 \neq s_3 \neq s_1)$$

$$u_z = \sum_{i=1}^{3} \alpha_{i1} \frac{\partial \psi_i}{\partial z_i} \qquad \phi = \sum_{i=1}^{3} \alpha_{i2} \frac{\partial \psi_i}{\partial z_i} \tag{4}$$

where s_1, s_2, and s_3 are the three roots of the characteristic equation, which is related to the following equation.

$$as^6 - bs^4 + cs^2 - d = 0 \tag{5}$$

where

$$a = c_{44}\left(e_{33}^2 + c_{33}\chi_{33}\right),$$

$$b = c_{33}\left[c_{44}\chi_{11} + \left(e_{15} + e_{31}\right)\right] + \chi_{33}\left[c_{11}c_{33} + c_{44} - \left(c_{13} + c_{44}\right)\right] + e_{33}\left[2c_{44}e_{15} + c_{11}e_{33} - 2\left(c_{13} + c_{44}\right)\left(e_{15} + e_{31}\right)\right]$$

$$c = c_{44}\left[c_{11}\chi_{33} + \left(e_{15} + e_{31}\right)\right] + \chi_{11}\left[c_{11}c_{33} + c_{44} - \left(c_{13} + c_{44}\right)\right] + e_{15}\left[2c_{11}e_{33} + c_{44}e_{15} - 2\left(c_{13} + c_{44}\right)\left(e_{15} + e_{31}\right)\right]$$

$$d = c_{11}\left(e_{15}^2 + c_{44}\chi_{11}\right).$$

In three roots of (5), s_1 is assumed to be a positive real number; s_2 and s_3 are either positive real number or a pair of conjugate complex roots with positive real parts.

1.1 Boundary Integral Equation

Based on the Somigliana equation, the boundary integral formulation is expressed as follows [8]:

$$\mathbf{C}(d)\mathbf{u}(d) = \int_{\Gamma} \mathbf{U}^*(d,x)\mathbf{t}(x)d\Gamma - \int_{\Gamma} \mathbf{T}^*(d,x)\mathbf{u}(x)d\Gamma \tag{6}$$

where C is the coefficient matrix, which depends on shape of the boundary Γ, and the general displacement vector u, and surface traction vector t are as follows:

$$\mathbf{u} = \{u \quad v \quad w \quad -\phi\}^T, \ \mathbf{t} = \{t_x \quad t_y \quad t_z \quad -\omega\}^T$$

and two matrices U* and T* composed of fundamental solution are:

$$\mathbf{U}^* = \begin{bmatrix} u_{11}^* & u_{12}^* & u_{13}^* & \phi_1^* \\ u_{21}^* & u_{22}^* & u_{23}^* & \phi_2^* \\ u_{31}^* & u_{32}^* & u_{33}^* & \phi_3^* \\ u_{41}^* & u_{42}^* & u_{43}^* & \phi_4^* \end{bmatrix}, \ \mathbf{T}^* = \begin{bmatrix} t_{11}^* & t_{12}^* & t_{13}^* & \omega_1^* \\ t_{21}^* & t_{22}^* & t_{23}^* & \omega_2^* \\ t_{31}^* & t_{32}^* & t_{33}^* & \omega_3^* \\ t_{41}^* & t_{42}^* & t_{43}^* & \omega_4^* \end{bmatrix}$$

where u_{ij}^* and t_{ij}^* (i, j = 1, 2, 3) are displacements and surface tractions, respectively at a field point x in the j coordinate direction due to a unit load acting in the x_i directions at a load point d, u_{4j}^* and t_{4j}^* (j = 1, 2, 3) are displacement components and surface tractions, respectively, at a field point x in the j coordinate direction due to a unit electric charge at d. ϕ_i^* and ω_i^* (i = 1, 2, 3) are electric potentials and surface charges, respectively, at a field point x due to a unit load acting in the x_i directions at a load point *d*, and ϕ_4^* and ω_4^* are electric potential and surface charge, respectively at a field point *x* due to a unit electric charge at *d*.

If the boundary is discretized with an eight-node isoparametric quadratic element, then the boundary integral equation is written as follows:

$$\mathbf{C}(d)\mathbf{u}(d) + \sum_{e\in\Gamma}\sum_{i=1}^{8}\int_{-1}^{1}\int_{-1}^{1}\mathbf{T}^*\mathbf{u_i}N_i|J|d\zeta d\eta = \sum_{e\in\Gamma}\sum_{i=1}^{8}\int_{-1}^{1}\int_{-1}^{1}\mathbf{U}^*\mathbf{t_i}N_i|J|d\zeta d\eta \quad (7)$$

where N_i is the shape function, *J* is Jacobian matrix, u_i and t_i represent the displacement and surface traction at node, and *C* is the coefficient matrix.

1.2 Eigenequation

An eigenequation based on the finite element method (FEM) was used to analyze the singularity at the singular point in piezo-ceramics bonded joint. In the formulation of FEM, a spherical coordinate system with the origin at a singular point is introduced, and displacement within a sphere of radius r_o in the singular field is expressed using the characteristic root *p*, which is related to the order of singularity. The surface of the sphere is divided into mesh.

The eigenequation was formulated for determining the order of stress singularity as follows [2]:

$$\left(p^2[\mathbf{A}]+p[\mathbf{B}]+[\mathbf{C}]\right)\{\mathbf{U}\}=\{0\} \qquad (8)$$

where

$\{\mathbf{U}\}=\{u_r \quad u_\theta \quad u_\phi \quad \psi\}^T$, and

$$[\mathbf{A}]=\sum_s\left([\mathbf{k}_a]-[\mathbf{k}_{sa}]\right),\ [\mathbf{B}]=\sum_s\left([\mathbf{k}_b]-[\mathbf{k}_{sb}]\right),$$

$$[\mathbf{C}]=\sum_s\left([\mathbf{k}_c]-[\mathbf{k}_{sc}]\right)$$

Here, *p* represents the characteristic root, which is related to the order of singularity, λ as λ = 1-p. [A], [B] and [C] are matrices composed of material properties, and {U} represents the elastic displacement and electric potential vector.

$$[\mathbf{k}_a]=\int_{-1}^{1}\int_{-1}^{1}[\mathbf{B}_a]^T[\mathbf{D}][\mathbf{B}_a]\sin\theta|J_1|\,d\xi d\eta$$

$$[\mathbf{k}_b]=\int_{-1}^{1}\int_{-1}^{1}\left([\mathbf{B}_a]^T[\mathbf{D}][\mathbf{B}_b]+[\mathbf{B}_b]^T[\mathbf{D}][\mathbf{B}_a]\right)\sin\theta|J_1|\,d\xi d\eta$$

$$[\mathbf{k}_c]=\int_{-1}^{1}\int_{-1}^{1}[\mathbf{B}_b]^T[\mathbf{D}][\mathbf{B}_b]\sin\theta|J_1|\,d\xi d\eta$$

$$[\mathbf{k}_{sa}]=2\int_{-1}^{1}\int_{-1}^{1}[\mathbf{H}]^T[\mathbf{SD}][\mathbf{B}_a]\sin\theta|J_1|\,d\xi d\eta$$

$$[\mathbf{k}_{sb}]=\int_{-1}^{1}\int_{-1}^{1}\left(2[\mathbf{H}]^T[\mathbf{SD}][\mathbf{B}_b]+[\mathbf{H}]^T[\mathbf{SD}][\mathbf{B}_a]\right)\sin\theta|J_1|\,d\xi d\eta$$

$$[\mathbf{k}_{sc}]=\int_{-1}^{1}\int_{-1}^{1}[\mathbf{H}]^T[\mathbf{SD}][\mathbf{B}_b]\sin\theta|J_1|\,d\xi d\eta \qquad (9)$$

Equation (8) now expressed as follows:

$$(-p[\mathbf{B}]-[\mathbf{C}])\{\mathbf{U}\}=p^2[\mathbf{A}]\{\mathbf{U}\}$$

Finally, letting $\{\mathbf{V}\}=p\{\mathbf{U}\}$, the characteristic equation can be transformed into the standard Eigen problem.

$$\begin{bmatrix} -[\mathbf{A}]^{-1}[\mathbf{B}] & -[\mathbf{A}]^{-1}[\mathbf{C}] \\ [\mathbf{I}] & [0] \end{bmatrix}\begin{Bmatrix}\{\mathbf{V}\}\\ \{\mathbf{U}\}\end{Bmatrix}=p\begin{Bmatrix}\{\mathbf{V}\}\\ \{\mathbf{U}\}\end{Bmatrix} \quad (10)$$

III. RESULT AND DISCUSSION

Figure 1 represents a model for 3D piezo-ceramic bonded structure used in the present analysis. The dimensions of material 1 are 10mm×10mm×$h$mm and material 2 are 20mm×20mm×5mm. The displacement and electric potential in the z-direction on the bottom in the model is fixed. The model is subjected to a uniform tension (1 MPa) and electric displacement (1 C/m2) whose poling direction is parallel to the z-axis. BaTiO3 and LiNbO3 are used for Materials 1 and 2, respectively, in the analysis. Fig. 2 represents the geometry of a typical case where a singular stress occurs at the point o. The region surrounding the singular point is divided into a number of quadratic elements with a summit o, with each element being located in spherical coordinates r, θ, and ϕ by its nodes 1 to 8.

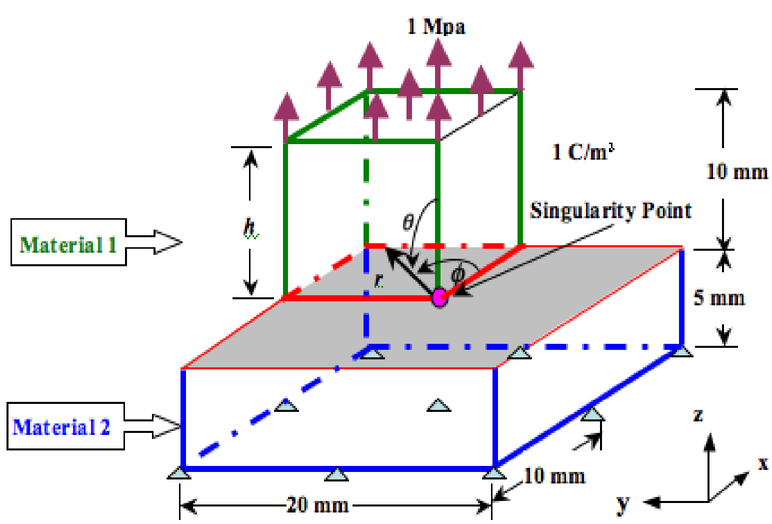

Fig.1. Model of analysis for piezo-ceramic bonded joints.

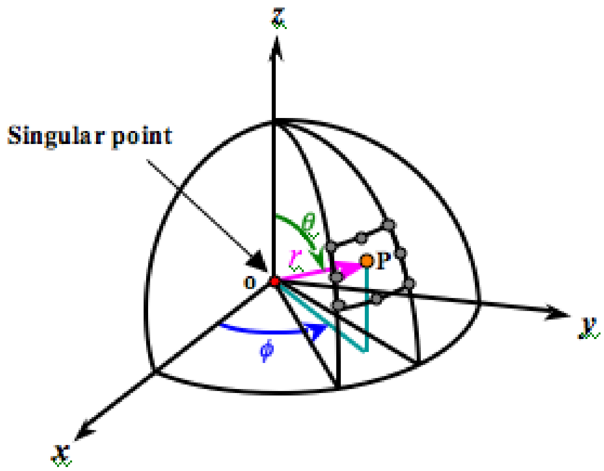

Fig.2. Element geometry and natural coordinates at a free edge singular point

Table1. Material properties of piezo-ceramic materials

Material	Elastic Constant, 10^{10} N/m^2				
	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}
$BaTiO_3$	15.0	6.60	6.60	14.6	4.4
$LiNbO_3$	20.3	5.73	7.52	24.2	5.95
Material	Piezoelectric Constant, C/m^2			Dielectric Constant, 10^{-10} C/Vm	
	e_{31}	e_{15}	e_{33}	χ_{11}	χ_{33}
$BaTiO_3$	-4.35	11.4	17.5	128.38	150.52
$LiNbO_3$	0.23	3.76	1.33	3.92	2.74

The order of singularity λ, at the vertex for the above model is calculated by eigenanalysis method. Solving eigenequation yields many roots p and eigenvectors are obtained. However, if the root p is within the range of $0 < p < 1$, this fact indicates that the stress field has singularity. The values of the order of singularity at the singularity corner are 0.6057, 0.3180, 0.1944, and 0.0711. The boundary condition in BEM analysis is agreed with the first value of the order of stress singularity for stress and electric displacement. Therefore, the first value of the order of stress singularity at the vertex is used in intensity of stress and electric displacement singularity calculation.

The distributions of stress and electric displacement in the singular field for the piezo-ceramic bonded structure are obtained using the boundary element method. The distributions of stress and electric displacement in the singular field against radial distance, *r* for various thicknesses of upper material are shown in Figs. 4 to7. It is shown from the figures that the stress and electric displacement increase with the thickness of upper material. All the figures show that the stress and electric displacement is larger at the vertex of the interface of the piezo-ceramics bonded joint. Therefore, there is a possibility to debond and delamination at the corner of the joint.

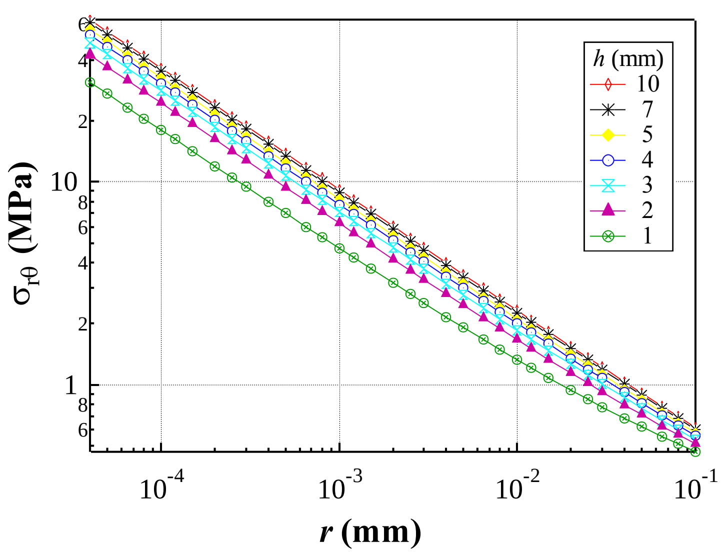

Fig. 4 Distributions of stress against r

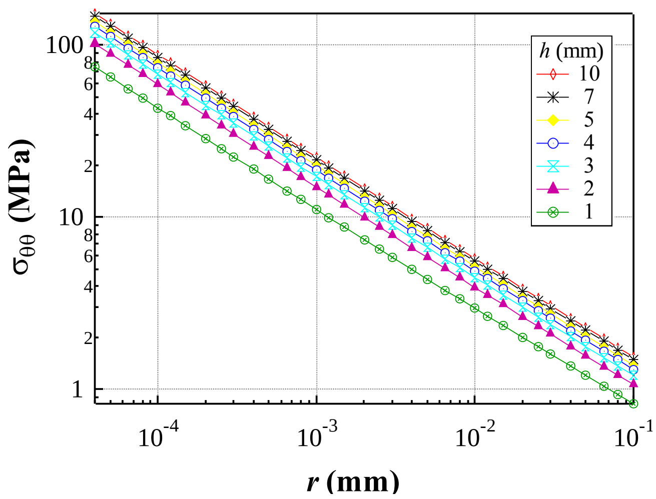

Fig. 5 Distributions of stress $\sigma_{r\theta}$ ***against r***

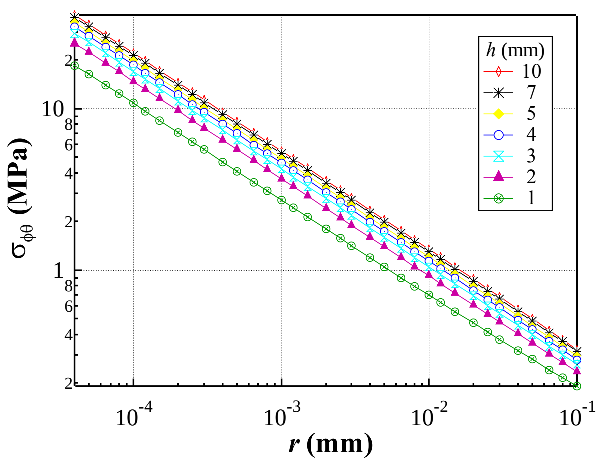

Fig.6. Distributions of stress $\sigma_{\phi\theta}$ ***against r***

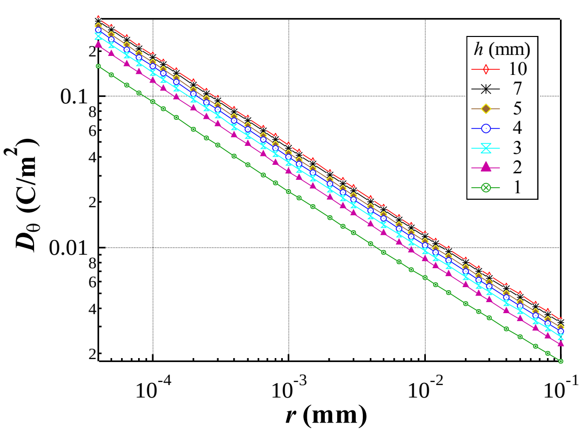

Fig.7 Distributions of electric displacement D_θ ***against r***

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Figs. 8 to 11 show the distributions of stress and electric displacement with respect to the angle ϕ at θ= 90° for various thicknesses of upper material. The stress singularity lines are at the free edge (ϕ= 0° and 90°) in the joint. . It is also shown from the figures that the stress and electric displacement increase with the thickness of upper material. All the figures show that the values of angular function of stress and electric displacement increase rapidly near the free edge than the inner portion of the joint. So, there is another possibility of debonding and delamination occurs near the interface edge of the piezo-ceramics bonded joint.

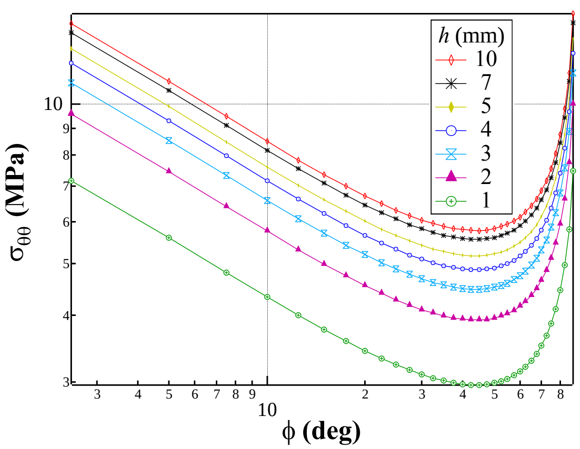

Fig. 8 Distributions of stress $\sigma_{\theta\theta}$ ***against angle,*** ϕ

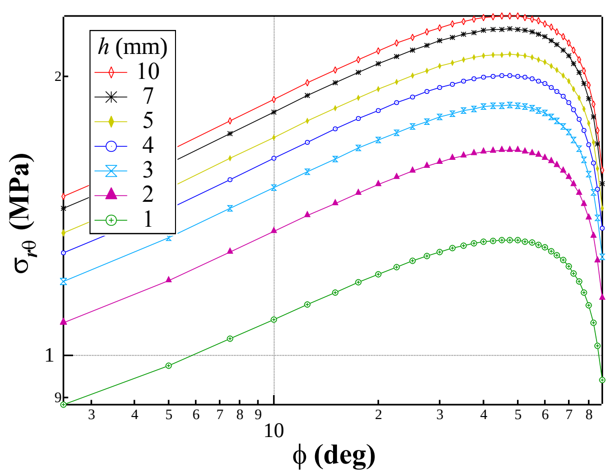

Fig. 9 Distributions of stress $\sigma_{r\theta}$ ***against angle,*** ϕ

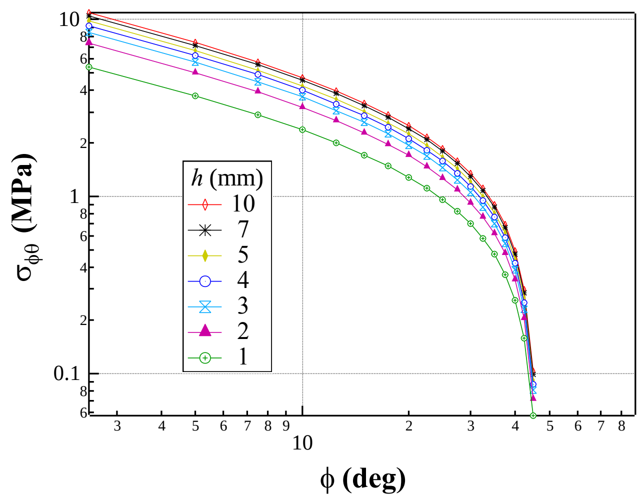

Fig. 10 Distributions of stress $\sigma_{\phi\theta}$ against angle, ϕ

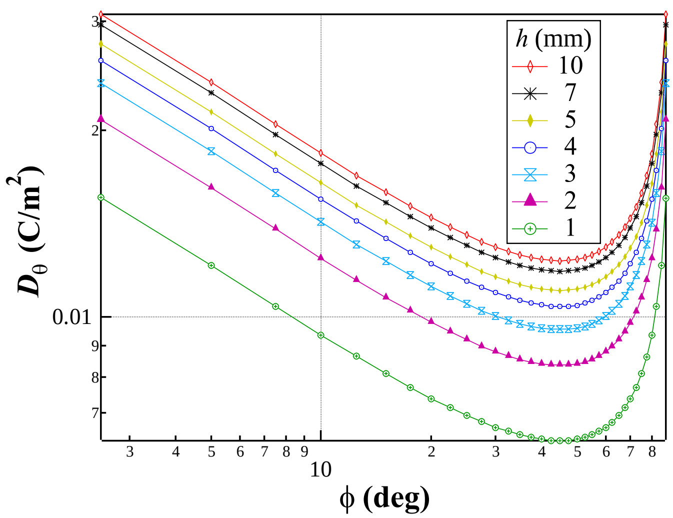

Fig. 11 Distributions of electric displacement D_θ against angle, ϕ

Figs. 12 and 13 show the distributions of normalized stresses $\sigma^*_{\theta\theta}$, $\sigma^*_{r\theta}$, $\sigma^*_{\phi\theta}$ and electric displacement, D^*_θ with respect to the angle ϕ at θ = 90° with r = 0.01mm. The stresses $\sigma^*_{\theta\theta}$, $\sigma^*_{r\theta}$ and electric displacement, D^*_θ are normalized by the values at $\phi = \pi/4$ and $\sigma^*_{\phi\theta}$ is normalized by the values at ϕ =15°. The solid line indicates the fitting of the angular functions of stress and electric displacement curves. The values of coefficients in angular functions are determined for singularity line, λ_1 = 0.456.

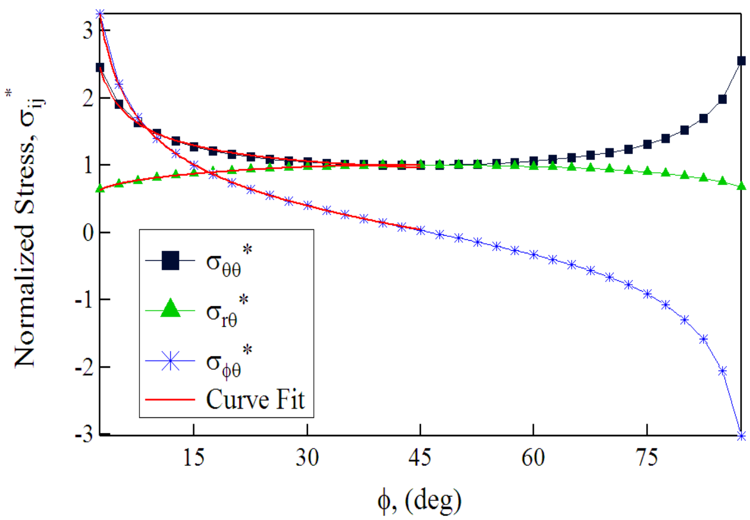

Fig. 12 Distributions of normalized stress against angle, ϕ

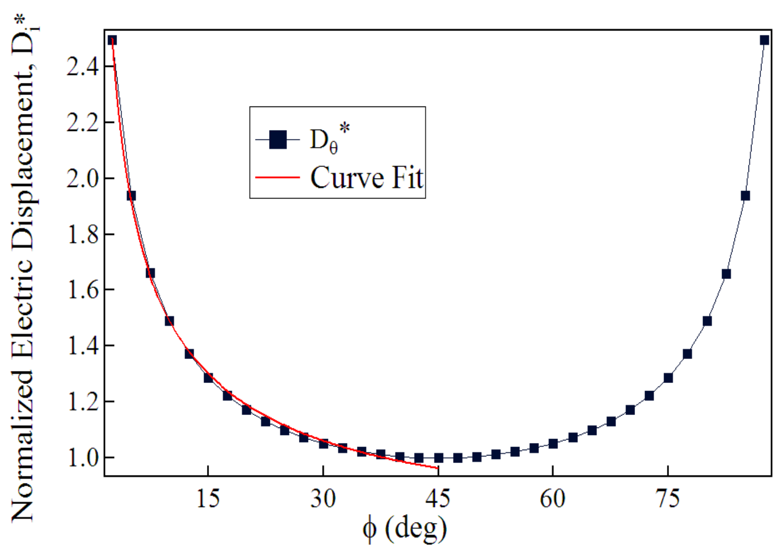

Fig. 13 Distributions of normalized electric displacement against angle, ϕ

The intensities of singularity in the r- and the ϕ-directions are determined from the distributions of stress and electric displacement using a least square method. Finally, the intensities of singularity at a vertex for stress and electric displacement are determined by varying the upper material thickness, *h*. The intensities of singularity at the vertex for stress and electric displacement are calculated by using the following relation.

$$K_{1ij}^{3D} = K_{1ij} L_{1ij}, \quad M_{1i}^{3D} = M_{1i} N_{1i} \quad (11)$$

Figs. 14 and 15 show a relationship between the intensities of singularity at the vertex of stress and electric displacement and the thickness of upper material. The intensity of singularity is determined by using (11). The intensity of singularity for stress and electric displacement increases with the thickness of upper material. The values of K_{1ij}^{3D} and M_{1i}^{3D} include the effect of the vertex singularity in the r-direction and line singularity in the ϕ-direction. It is found from the analysis that the value of $K_{1\theta\theta}^{3D}$ is larger than the value of $K_{1r\theta}^{3D}$ and $K_{1\phi\theta}^{3D}$.

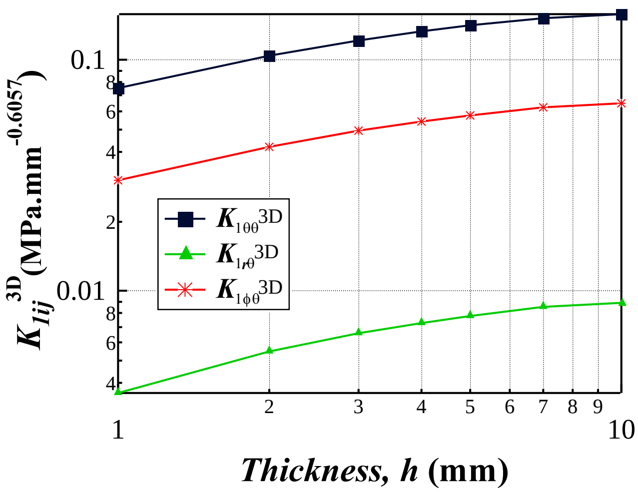

Fig. 14 Distributions of intensity of singularity K_{1ij}^{3D} against thickness, h

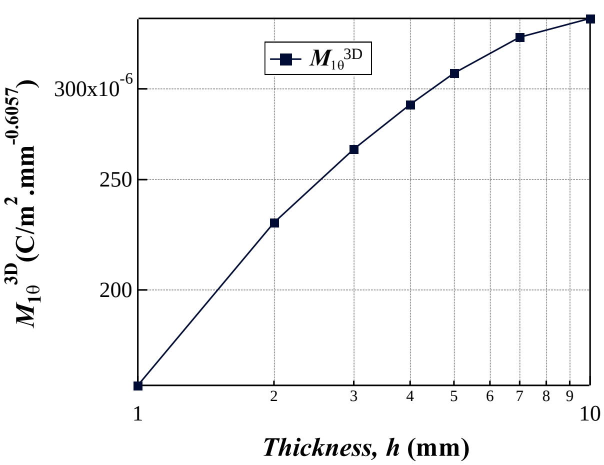

Fig. 15 Distributions of intensity of singularity $M_{1\theta}^{3D}$ against thickness, h

CONCLUSION

The order and the intensity of singularity that characterizes a singular field at a vertex in transversely isotropic oiezo-ceramics bonded joints were investigated using eigenanalysis method and a boundary element method. The order of singularity for stress and electric displacement was calculated in eigenanalysis based on a finite element method. The order of singularity at a vertex and at a point in singularity line was calculated. The vertex singularity has a larger value than the line singularity. The distributions of stress and electric displacement with respect to radial distance and angle ϕ for various thickness of upper material were calculated in boundary element analysis. The stress and electric displacement increases with the thickness of upper material. The intensities of singularity were calculated by fitting the stress and electric displacement curves with the help of the result of eigen analysis. Finally, the intensities of singularity for stress and electric displacement were determined. The intensity of singularity also increases with the thickness of upper material. Therefore, there is a highest possibility to de bond and delamination may occur near the vertex of thick piezo-ceramics bonded joint.

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