

Effect of Slant Angle on Piezoelectric Bonded Joint

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ABSTRACT

Piezoelectric materials have great influence on the development of smart structures because of its high functionality. Piezoelectric materials are widely used to make sensors and transducers as a result of their inherent straight and opposite piezoelectric effects developed between mechanical and electric deformation. In this paper the singular stress field and electric displacement distribution near the vertex along interface edge of piezoelectric bonded joints is analyzed. Abaqus FEA software is used for stress singularity field analysis of piezoelectric dissimilar material joints. From the numerical result, it is obtained that displacement, electric potential, stress and electric displacement development near the vertex and interface edge is the maximum for model slant 75° and the minimum for model slant 45°.

Keywords: Piezoelectric material, Electric displacement, Slant angle, Singular stress components.

1. Introduction

Piezoelectric materials are used to make sensors and transducers as a result of their inherent straight and opposite piezoelectric effects developed between mechanical and electric deformation. In electronic industry, these have gathered more concentration due to high functionality.

Enormous researches based on bonded joint can be found. Thermal stress has been analyzed by D. Munz et al. [1] in ceramic-metal joints by varying interlayer thickness where thermal and electrical load is applied by them. Somnath Somadder and Md. Shahidul Islam [2] investigated stress and displacement field of cylinder subjected to thermo-mechanical loadings by using finite element method. Hideo Koguchi [3] analyzed stress singularity at three dimensional bonded joints which is considered a novel approach in bonded joint analysis. G. Pamnani et al. [4] employed XFEM to analyze cracks in tri-material piezoelectric bonded joints. Hideo Koguchi and Koki Yokoyama [5] analyzed singular stress field with a crack near the vertex of the interface since a plastic zone occurs near the crack tip region.

2. Formula of analysis

The mechanical equilibrium equation is given by [6]

$$\int_V \sigma : \delta \varepsilon dV = \int_S t \cdot \delta u dS + \int_V f \cdot \delta u dV \quad (1)$$

where $\delta \varepsilon = \text{sym} \frac{\partial \delta u}{\partial x}$, and δu is taken as arbitrary vector field and its every component is continuous.

The electrical flux conservation equation is written as,

$$\int_V q \cdot \delta E dV = \int_S q_s \delta \varphi dS + \int_V q_v \delta \varphi dV \quad (2)$$

where $\delta E = \text{sym} \frac{\partial \delta \varphi}{\partial x}$, and $\delta \varphi$ is an arbitrary scalar field which is considered to be continuous

In Abaqus the following constitutive equations are used [7]

$$\sigma_{ij} = D_{ijkl}^E \varepsilon_{kl} - e_{mij}^\varphi E_m \quad (3)$$

$$q_i = e_{ijk}^\varphi \varepsilon_{jkl} + D_{ij}^{\varphi(\varepsilon)} E_j \quad (4)$$

D_{ijkl} indicates the material stiffness; and e_{ijk}^φ denotes piezoelectric constant; and D_{ij}^φ is the dielectric constant.

The finite element discretization is conducted which estimates the following solution [8]

$$\begin{aligned} u_c &= N_u^T \cdot u \\ V_c &= N_V^T \cdot V \end{aligned} \quad (5)$$

where N_u denotes displacement shape functions, N_V indicates electrical potential shape function.

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Using (5), for the strain and electric field it can be written as,

$$S = B_u \cdot u \quad (6)$$

$$E = B_v \cdot V \quad (7)$$

where,

$$B_u = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}^T, \quad (8)$$

$$B_v = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix}^T$$

The coupled finite element matrix equation is obtained by utilizing the variational principle and the finite element discretization,

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{V} \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{V} \end{bmatrix} + \begin{bmatrix} K & K_z \\ K_z^T & K_d \end{bmatrix} \begin{bmatrix} u \\ V \end{bmatrix} = \begin{bmatrix} F \\ L \end{bmatrix} \quad (9)$$

where,

$$\text{Structural Mass is indicated by } M = \int \rho N_u N_u^T dV$$

$$\text{Structural Stiffness is denoted by } K = \int B_u^T C B_u dV$$

Piezoelectric coupling matrix is expressed as follows

$$K_z = \int B_u^T e B_v dV$$

Dielectric conductivity is denoted by

$$K_d = \int B_v^T \varepsilon B_v dV$$

3. Model of analysis

Abaqus 6.14 finite element software is used for the generation of finite element model.

3.1 Physical aspects of the model

The dimensions of the model are 20x20x20 mm. By taking advantage of symmetry one-fourth of the model will be analyzed. This is done to reduce computational time with proper mesh size.

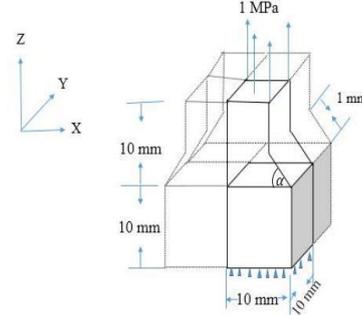


Fig.2 Model for finite element analysis

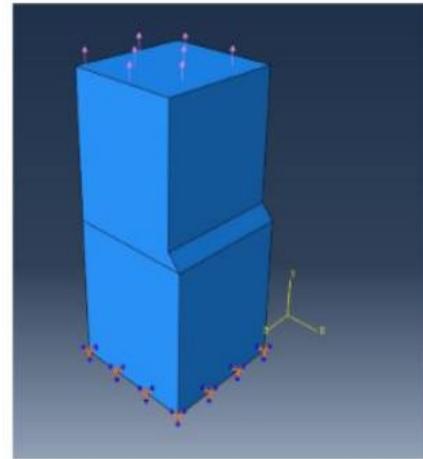


Fig.3 Model showing the boundary condition of analysis.

In this analysis the lower surface is fixed and mechanical load is applied on the upper surface. After creating the parts and assigning material properties the parts are assembled together.

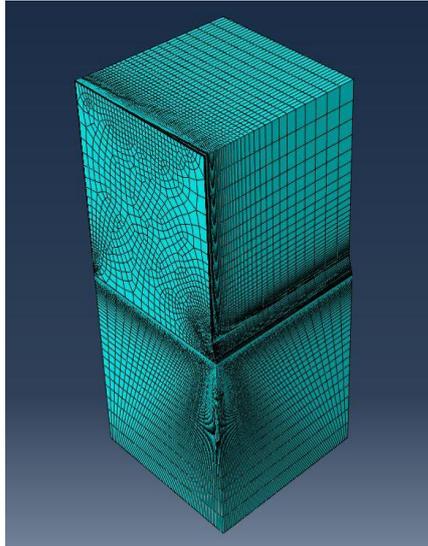


Fig.4 Mesh of the model.

Single bias has been used at the vertex of the joint so there is more element near the vertex than the other regions. As this is a piezoelectric analysis; 8 node linear piezoelectric brick element C3D8E is used in this analysis.

3.1 Material Properties

In case of piezoelectric and isotropic bonded joint model PZT 5H is used as upper material and Resin is used as lower material. For piezoelectric and piezoelectric bonded joint model PZT 5H is used as upper material and PZT 5A is used as lower material. The material properties of piezoelectric materials PZT 5H, PZT 5A, and Resin is shown in the Table 1.

Table 1 Elastic properties for PZT 5H, PZT 5A and Resin

Elastic Constant	PZT 5H	PZT 5A	Resin
E_1	60.61e9	54.054e9	E = 2.74e9 $\nu = 0.38$
E_2	60.61e9	54.054e9	
E_3	48.31e9	48.31e9	
ν_{12}	0.289	0.41	
ν_{13}	0.512	0.41	
ν_{23}	0.512	0.41	
G_{12}	23.5e9	19.14e9	
G_{13}	23e9	19.48e9	
G_{23}	23e9	19.48e9	

Table 2 Dielectric properties for PZT 5H and PZT 5A

Dielectric Constant	PZT 5H	PZT 5A
D_{11}	1.505e-8	1.72e-8
D_{22}	1.505e-8	1.72e-8
D_{33}	1.301e-8	1.72e-8

The piezoelectric constants of PZT 5H, PZT 5A are selected depending upon various parameter requirements.

4. Numerical Result and discussion

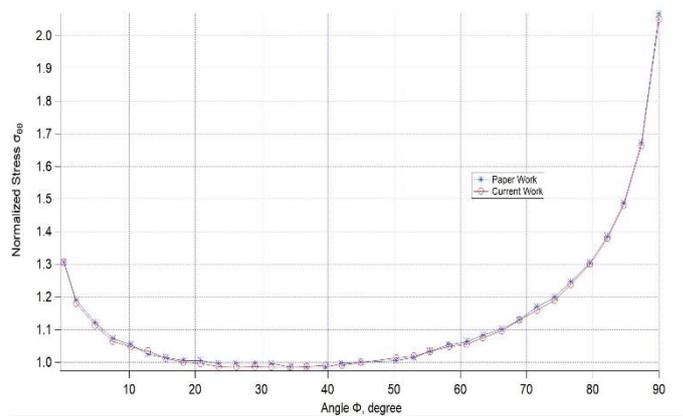


Fig.5 Accuracy verification of present analysis.

For the verification of this research work the result obtained from running the commercial code using Abaqus FEA software is compared to the result of a journal paper [9]. The results obtained in the current work is in good agreement with the published results.

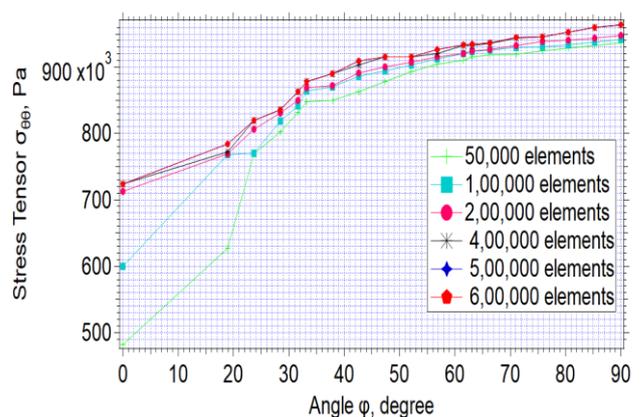


Fig.6 Mesh sensitivity analysis of present work.

From mesh sensitivity analysis it can be observed that the results obtained using 5,00,000 elements are same as the results obtained using 600000 elements. In order to save the time of calculation the optimum number of elements for further analysis is 5,00,000 elements.

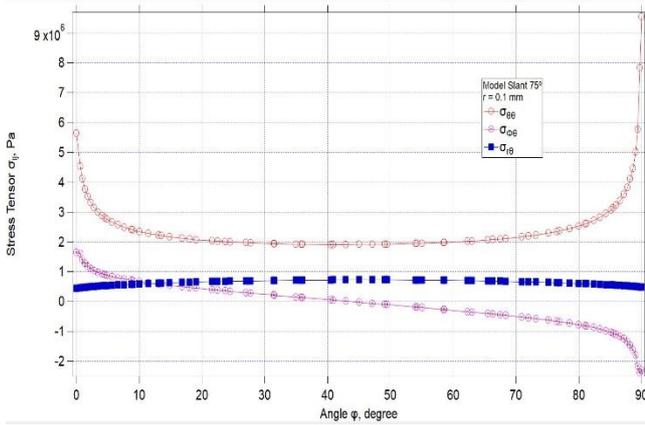


Fig.7 Stress variation on the interface at $r = 0.1$ mm of piezoelectric (PZT 5H) and isotropic (Resin) bonded joint (model slant 75°).

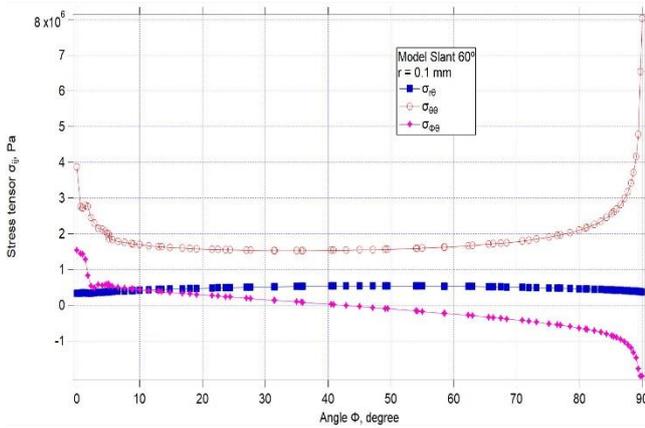


Fig.8 Stress variation on the interface at $r = 0.1$ mm of piezoelectric (PZT 5H) and isotropic (Resin) bonded joint (model slant 60°).

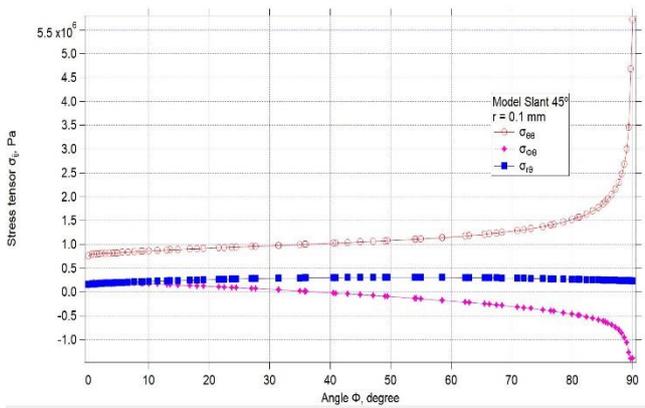


Fig.9 Stress variation on the interface at $r = 0.1$ mm of piezoelectric (PZT 5H) and isotropic (Resin) bonded joint (model slant 45°).

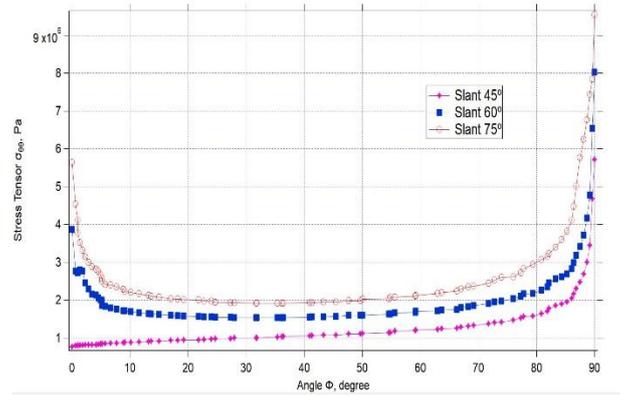


Fig.10 Stress variation on the interface at $r = 0.1$ mm of piezoelectric (PZT 5H) and isotropic (Resin) bonded joint for different slant angle.

From figure 7, 8,9,10 it is clear that stress near the flat side surface is larger than the slanted surface. Figure 9 denotes that large stress development near the vertex and interface edge is prevented by using slanted side surface. From the numerical result, it is obtained that displacement, electric potential, stress and electric displacement development near the vertex and interface edge is the maximum for model slant 75° and the minimum for model slant 45° . So, slant 45° model is more reliable for operation.

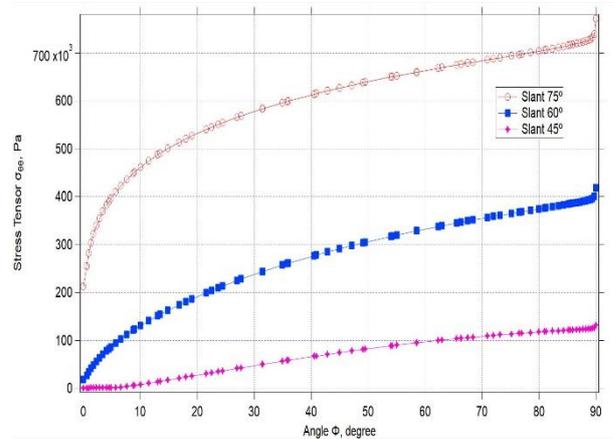


Fig.11 Stress variation on the interface at $r = 0.1$ mm of piezoelectric (PZT 5H) and piezoelectric (PZT 5A) bonded joint for different slant angle.

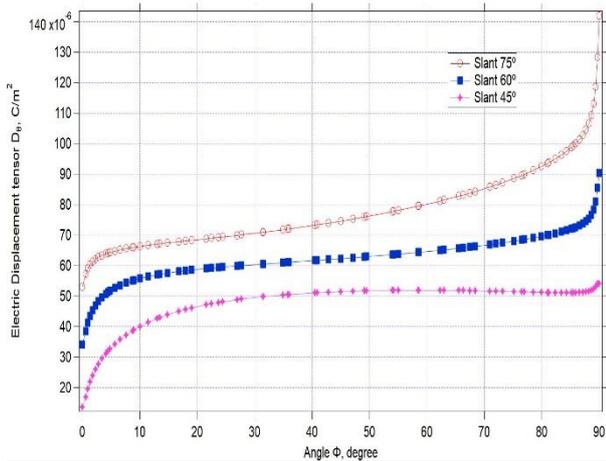


Fig.12 Electric displacement variation on the interface at $r = 0.1$ mm of piezoelectric (PZT 5H) and piezoelectric (PZT 5A) bonded joint for different slant angle.

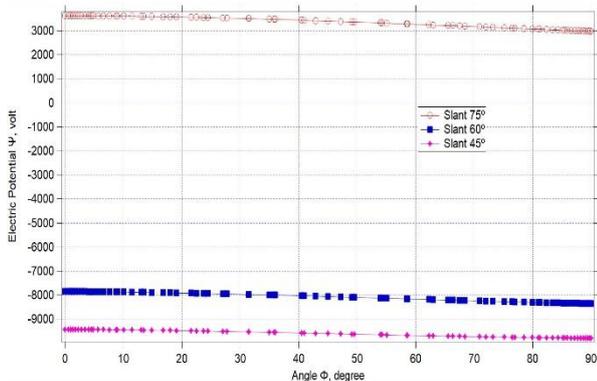


Fig.13 Electric potential variation on the interface at $r = 0.1$ mm of piezoelectric (PZT 5H) and piezoelectric (PZT 5A) bonded joint for different slant angle.

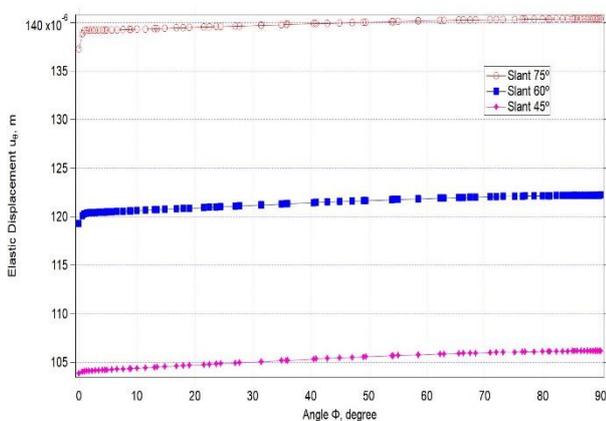


Fig.14 Elastic displacement variation on the interface at $r = 0.1$ mm of piezoelectric (PZT 5H) and isotropic (Resin) bonded joint for different slant angle.

From figure 11, 12, 13,14 it is clear that large stress, electric displacement, electric potential, elastic displacement development near the vertex and interface edge is prevented by using slanted side surface. From the numerical result, it is obtained that displacement, electric potential, stress and electric displacement development near the vertex and interface edge is the minimum for model slant 45° . So, slant 45° model is more reliable for operation.

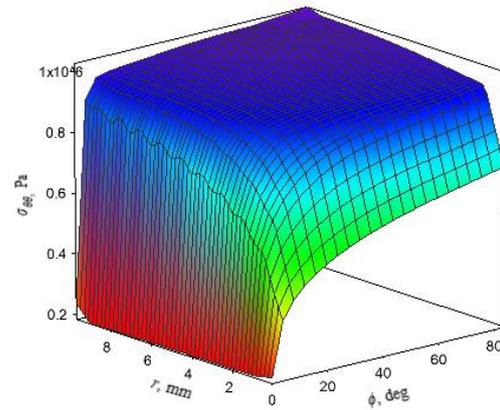


Fig.15 Variation of stress tensor $\sigma_{\theta\theta}$ against radial distance r and angle ϕ on the interface of piezoelectric dissimilar material joint (model slant 75°).

Figure 15 shows the distribution of stress component $\sigma_{\theta\theta}$ in r - ϕ plane at the interface. It is clear from the figure that normal $\sigma_{\theta\theta}$ stress is higher at $\phi = 90^\circ$ and low at $\phi = 0^\circ$. This indicates that slanted side surface reduces stress. The value of stress is high at $r = 0$ and value of stress decreases as the radial distance increases.

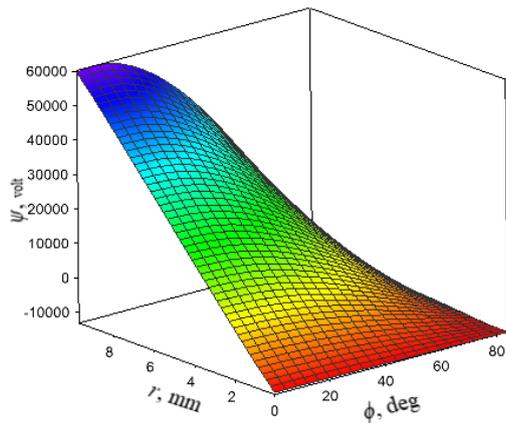


Fig.16 Variation of electric potential ψ against radial distance r and angle ϕ on the interface of piezoelectric dissimilar material joint (model slant 75°).

From figure 16 it can be observed that electric potential varies sharply along radial direction but the change of electric potential is not so sharp along ϕ direction.

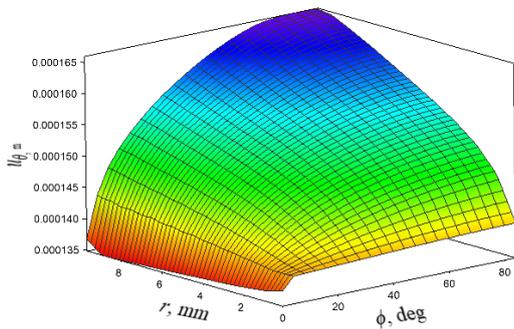


Fig.17 Variation of elastic displacement u_θ against radial distance r and angle ϕ on the interface of piezoelectric dissimilar material joint (model slant 75°).

From figure 17 it can be observed that elastic displacement is higher at $\phi = 90^\circ$ and low at $\phi = 0^\circ$. This indicates that slanted side surface reduces elastic displacement. The value of elastic displacement is high at $r = 0$ and value of elastic displacement decreases as the radial distance increases.

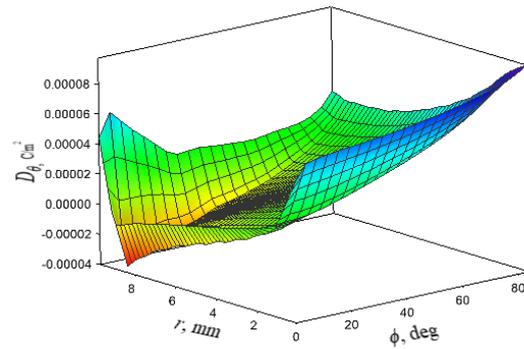


Fig.18 Variation of electric displacement D_θ against radial distance r and angle ϕ on the interface of piezoelectric dissimilar material joint (model slant 75°).

From figure 18 it can be observed that electric displacement is higher at $\phi = 90^\circ$ and low at $\phi = 0^\circ$. This indicates that slanted side surface reduces electric displacement. The value of electric displacement is high at $r = 0$ and value of electric displacement decreases as the radial distance increases.

6. Conclusion

In the present work the effect of slanted side surface on the stress singular field at a vertex in 3D piezoelectric bonded joints was investigated using finite element analysis. Stress field, electric potential, electric displacement was determined by running a commercial code using Abaqus 6.14 FEA software. From the numerical results, the following conclusions can be drawn for the piezoelectric dissimilar material joints-

1. Maximum displacement, electric potential, stress and electric displacement occurs near the vertex and along the interface edge than other regions.
2. Displacement, electric potential, stress and electric displacement development near the vertex and interface edge is maximum for model slant 75° and minimum for model slant 45° .

8. References

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NOMENCLATURE

- σ – True stress, Pa
 t – Traction force, Pa
 q – Electric flux, $N\ m^2\ C^{-1}$
 u – Displacement, m
 V – Electric Potential, $N\ m\ C^{-1}$
 S – Strain
 ρ – Mass density, Kg/m^3