

ICMIEE20-160

## CFD Analysis of Natural Convection in a Triangular Cavity Filled with Nanofluid Having a Rectangular Heat Source at the Bottom

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### ABSTRACT

This CFD analysis explores the consequences of heat transfer by the process of free convection heat transfer inside a triangular cavity, which has an oblong heat source at the bottom. The entire cavity has been filled with nanofluid. The surfaces adjacent to the heat source have been assumed as an adiabatic wall. The two other faces of the triangle are treated as cold faces while maintaining the constant room temperature. In this investigation, two different types of nanofluid, Al<sub>2</sub>O<sub>3</sub>-water, and Cu-water have been used. This computational study has been carried out for Rayleigh numbers within the range of  $10^3 \leq Ra \leq 10^7$  and fraction of solid nanoparticle that ranges within  $0.0 \leq \phi \leq 0.1$ . The impact of parameters such as volume fraction of Al<sub>2</sub>O<sub>3</sub> and Cu nanoparticles in the water, heat source temperature & variation of Rayleigh number has been studied. Results exhibit that the average heat transfer rises with the increment of Rayleigh number. Besides, the effects of Rayleigh number in terms of avg. Nusselt number, isotherm lines, and stream function lines have been studied. The consequences show that the average heat transfer rate rises simultaneously with the volume fraction of particles. On the other hand, the average Nusselt number decreases instead. Differences between heat transfer rates among these two nanofluids have also been concluded.

Keywords: Triangular Cavity, CFD, Nusselt Number, Nanofluid, Rayleigh Number

### 1. Introduction:

The natural convection heat transfer process is broadly used in various designing applications because of its huge applications in geophysics, atomic reactor framework, cooling of hardware framework, close planetary system, so on. In the last few years, researchers have utilized numerous techniques for increasing heat-transfer coefficient. Nanofluid is one of those efficient mediums, which has higher heat conductivity rather than other fluids. It is one of the forms of nanotechnology that has become a topic of attraction because of its tremendous heat transfer performance in different sectors including cooling, defense, energy/power production, space, nuclear, microelectronics, and many other sectors. Numerous experiments of nanofluid have already been performed on its flow and characteristics of heat transfer [1]. Dogonchi studied the Cu-water nanofluid in three-sided cavities with a half-circle base that created Lorentz force [2]. In recent years, many researchers interested in cavities due to their numerous applications. Koca et al. explained the natural convection for a range of Prandtl numbers in a three-sided enclosure [3]. All of the equations were put together and resolved by using finite-difference process. The Prandtl number varied with the temperature. The natural convective heat transfer in the triangular cavity with a heater had numerically investigated by Varol et al. [4]. Bouabdallah [5] et al. investigated heat transfer relation

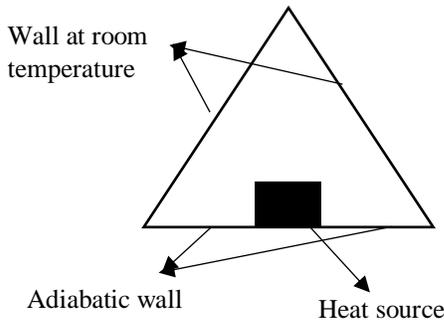
with variations of cavity aspect ratio. They also got that rate of heat transfer rises with the heat generation source height when the position of the heater must be in the center. He studied natural convection inside a uniform four-sided-shaped enclosure. It was found that avg. Nusselt number had raised along with the Rayleigh number. In another analysis, Bhardwaj et al. [6] observed the triangular (right) cavity for heat transfer as well as entropy generation in having an undulated wall and concluded that Nusselt number has expanded up to 53% for ripple on the left surface. In no-ripple case  $Da = 10^{-2}$  and  $Ra = 10^6$ . Shirvan et al. [7] studied natural convection in a wavy square perimeter. Heat transfer expanded at Rayleigh number (constant) for wave circumferences. And the Nusselt number also increases with nanoparticle volume fraction. From this above literature, concluded that several works have already been done on nanofluid to investigate the heat transfer enhancement technique. But the triangular cavity a having heat source at the bottom didn't evoke that much attention. Thus, this numerical analysis aims at examining heat transfer characteristics on a triangular-shaped enclosure having nanofluid and also heated from the bottom. 2 types of nanofluids have been used to perform this investigation and the effectiveness of those nanofluids on heat transfer enhancement has been compared. Besides, the impact of other parameters like volume fraction, Rayleigh number are inspected and graphically presented.

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## 2. Geometrical Model and Mathematical Formulation

The schematic diagram of a triangular cavity having a heat source underneath is shown in Fig.1. The entire cavity has been filled with nanofluid. The rectangular heat source of the enclosure kept at a fixed temp ( $T_h$ ) as well as fixed room temperature has been maintained by inclined walls ( $T_c$ ), which is lower than the heat source temp. The lower horizontal part is assumed to be an adiabatic wall. The enclosure made of Aluminium is presumed inaccessible. The water has been used as a working fluid in the cavity. The fluid flow inside is incompressible and has been created from the difference of temperature. It has been supposed to be laminar. The exchange of heat due to radiation and effects due to viscosity is considered ineffective. All the properties of the working fluid are treated as constant. The analysis has been done by ANSYS Fluent v16.2 software formed on the finite-volume method.



**Fig.1** Schematic diagram of the triangular enclosure

The assessment has governed by the continuity equation, the energy equation, and the Navier-Stokes equations. Those equations are written below-

### The Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (1)$$

### The Momentum Equation: (for both x and y directions)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} * \frac{\partial p}{\partial x} + \left[ \frac{\mu}{\rho} * \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \right] \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} * \frac{\partial p}{\partial y} + \left[ \frac{\mu}{\rho} * \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \right] - \frac{1}{\rho} * (\rho \beta)_{nf} * g(T - T_c) \quad (3)$$

### Energy Equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{q''}{\rho C_p} \quad (4)$$

The effective density ( $\rho_{nf}$ ), as well as the heat capacitance  $(\rho C_p)_{nf}$  for the nanofluid can be described as:

$$\rho_{nf} = (1 - \varphi)\rho_f + \rho_s \varphi \quad (5)$$

$$(\rho C_p)_{nf} = (\rho C_p)_f \times (1 - \varphi) + (\rho C_p)_s \times \varphi \quad (6)$$

Where  $\varphi$  denoted the volume portion of particles used in the mixture. The equation of thermal diffusivity for working nanofluid as follows,

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (7)$$

The equation that is used for thermal expansion co-efficient for working nanofluid have been concluded below,

$$\beta_{nf} = \beta_f \times (1 - \varphi) + \beta_s \times \varphi \quad (8)$$

The equation of dynamic viscosity for the nanofluid that has been given by Ogut E.B [8]is,

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \quad (9)$$

Nusselt number and Rayleigh number are defined as follows:

$$Nu = \frac{hL}{k} \quad (10)$$

$$Ra = \frac{g\beta\Delta T x^3}{\nu\alpha}$$

The Maxwell–Garnetts [9] model that is ideal for the effective thermal conductivity for the working nanofluid as follows:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2*k_f - 2\varphi(k_s - k_f)}{k_s + 2*k_f + 2\varphi(k_s - k_f)} \quad (11)$$

The usage of equation (11) is only confined for spherical nanoparticles only.

Boundary conditions applied are given below;

$$U(x,0) = 0 \text{ and } V(x,0) = 0 \quad 0 < x < S$$

$$U(x,S) = 0 \text{ and } V(x,S) = 0 \quad 0 < x < S$$

$$U(0,y) = 0 \text{ and } V(0,y) = 0 \quad 0 < y < S$$

$$U(S,y) = 0 \text{ and } V(S,y) = 0 \quad 0 < y < S$$

$$T(x,0) = T_h = 308 \text{ K}, T(x,S) = T_c = 298 \text{ K}$$

The dimensionless parameters are,

$$X = \frac{x}{s}, Y = \frac{y}{s}, U = \frac{uS}{\alpha}, V = \frac{vS}{\alpha}, \theta = \frac{T - T_c}{T_h - T_c}$$

**Table 1** Various thermo-physical properties of water [1], nano-fluid [1][10]

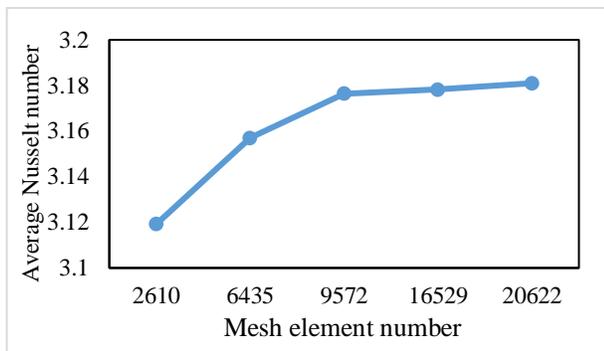
Properties	Pure Water	Aluminium Oxide	Copper
$\rho$ (kgm <sup>-3</sup> )	997.1	3880	8933
$C_p$ (J/kg-K)	4179	765	385
K(W/m-k)	0.6130	42.64	401
Thermal expansion coefficient (/k)	$21 \times 10^{-5}$	$8.46 \times 10^{-6}$	$9.4 \times 10^{-6}$

### 3. Mesh Generation and Validation

This paragraph comprises with mesh independence test and model validation, and lastly, the effects of volume portion of particles, variation of Rayleigh number, and heat source area has shown.

#### 3.1 Mesh Independence Test

The value of the average Nusselt number for different mesh sizes has been calculated. Five different structured meshes have been used in this analysis for a different number of elements to determine each solution's independence. All of the cavities have been filled with Water-Al<sub>2</sub>O<sub>3</sub> nanofluid having a 1% volume fraction of particles. Besides, each of the analysis has been tested for Ra=10<sup>5</sup>. Then the results have been plotted in the graph. It confirms from the plotting that a very slight change can be seen in average Nusselt number after the mesh elements 10000.



**Fig.2** Change of avg. Nusselt number with different mesh elements (Ra=10<sup>5</sup>, Al<sub>2</sub>O<sub>3</sub>. $\phi$  = 0.01)

### 3.2 Model Validation

In this section, the model has been validated with some previously published research papers to confirm this analysis has been done accurately. A two-dimensional rectangular cavity has been selected on which vertical walls are heated differentially, and the upper and bottom faces are considered to maintain adiabatic conditions. The average Nusselt number for the hot wall has been calculated from the present analysis. The result data has been compared with the reference papers and has been got close similarities for varying Rayleigh numbers.

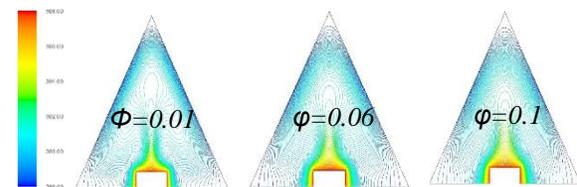
**Table 2** Disparities of present analysis with literature results in terms of Avg. Nusselt number

Ra	Nu			Percentage of error, %	
	Present Analysis	Dogonchi et al. [1]	Khanafar et al. [11]	1 <sup>st</sup> Case [1]	2 <sup>nd</sup> Case [11]
10 <sup>3</sup>	1.115	1.131	1.118	1.43	0.26
10 <sup>4</sup>	2.248	2.267	2.245	0.84	0.13
10 <sup>5</sup>	4.548	4.585	4.522	0.81	0.57
10 <sup>6</sup>	8.912	8.834	8.826	0.87	0.97

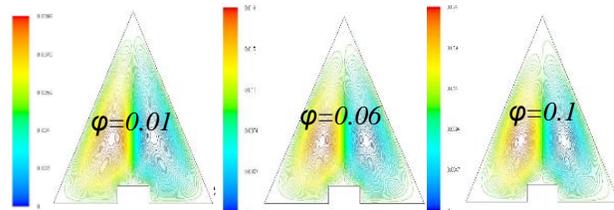
## 4. Results and Discussion

### 4.1 Consequence of particle volume fraction for different nano-fluid on natural convection

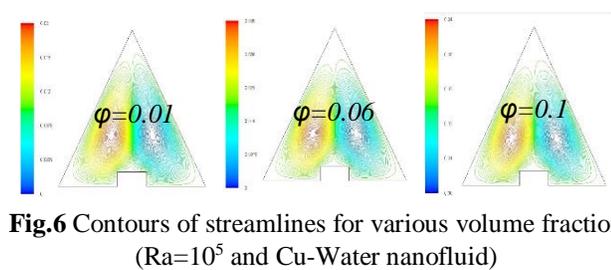
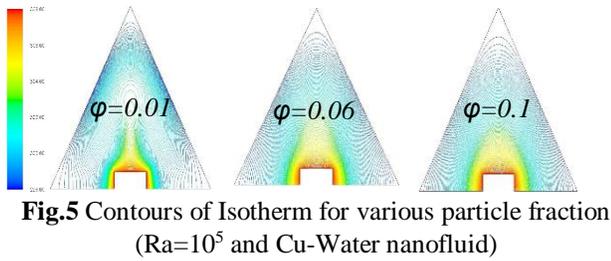
Contours of isotherms and stream functions have been shown below both for Al<sub>2</sub>O<sub>3</sub> and Cu nanofluid.



**Fig.3** Contours of Isotherm for various particle fraction (Ra=10<sup>5</sup> and Al<sub>2</sub>O<sub>3</sub>-Water nanofluid)

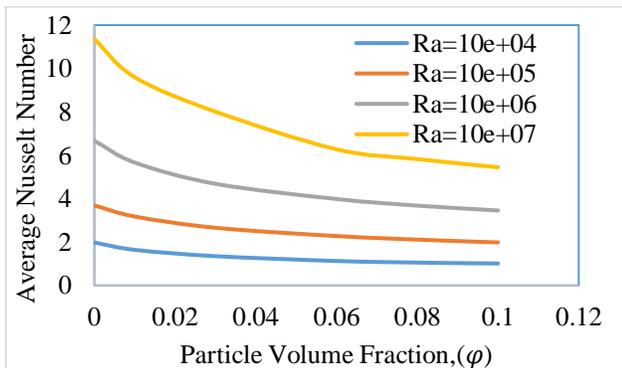


**Fig.4** Contours of Streamlines for various particle fraction (Ra=10<sup>5</sup> and Al<sub>2</sub>O<sub>3</sub>-Water nanofluid)

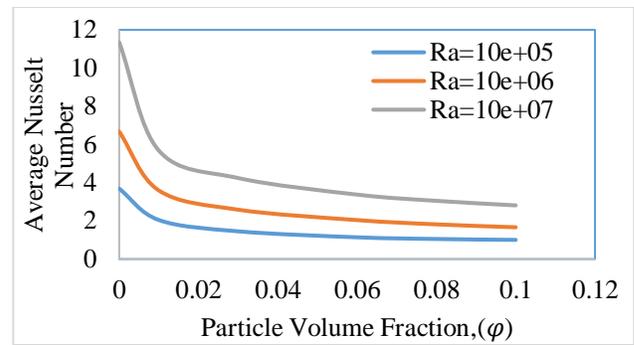


In case of isotherm (Fig.3 & Fig.5), contours are simultaneously changing with the volume fraction. It takes a stable shape in the higher volume fraction. Likewise, in the stream function (Fig.4 & Fig.6), the magnitude of stream function simultaneously increases with higher volume fraction as well as in higher volume fraction- it takes a good shape. Because of thermal conductivity, the change occurs.

In addition to those above contours, a graph has been plotted within the avg Nusselt number, volume fraction for Al<sub>2</sub>O<sub>3</sub> (Fig.7). It has been found that the avg. Nusselt number reduces when the nanosized particle volume fraction rises. For four particular Rayleigh numbers, it has also been noticed that the value of avg. Nusselt number rises with the rising Rayleigh number. The relation can be noticed from Fig.8 between avg Nusselt number, particle fraction for Cu-water nanofluid.

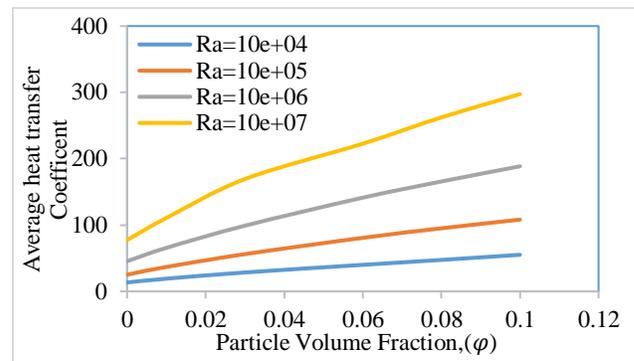


**Fig.7** Change of avg. Nusselt number with the changing volume fraction as well as Rayleigh number(Al<sub>2</sub>O<sub>3</sub>)

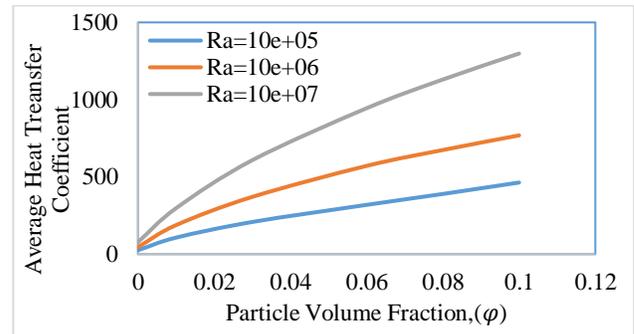


**Fig.8** Change of avg. Nusselt number with the changing volume fraction as well as Rayleigh number(Cu)

Here, it can be seen that the Nusselt no. reduces when the particle volume fraction increases. It has also been examined that the value of Nusselt number for Cu nanoparticles is less than Al<sub>2</sub>O<sub>3</sub> for the same Rayleigh number. In the case of Cu, the Nusselt number for Rayleigh number 10<sup>4</sup> is less than 1, which is not possible according to the definition of Nusselt number. So it has been avoided.



**Fig.9** Change of avg. heat transfer coefficient with the increasing particle fraction(Al<sub>2</sub>O<sub>3</sub>)



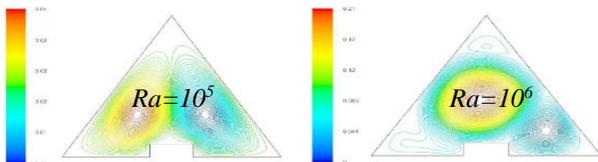
**Fig.10** Change of average heat transfer coefficient with the increasing particle fraction(Cu)

A relationship between the average heat transfer coefficient and volume fraction of Al<sub>2</sub>O<sub>3</sub> nanoparticles can be seen in (Fig.9). Here, the value of the heat transfer

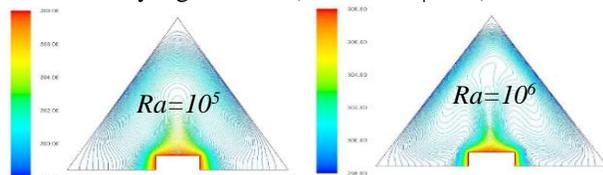
coefficient increases proportionally with the particle volume fraction and also with the increase of the Rayleigh number. Fig.10 also shows similar results for the Cu-water nanofluid. But the avg. heat transfer co-efficient for Cu is more than Al<sub>2</sub>O<sub>3</sub> for the same Rayleigh number. In the case of Cu, the value of Rayleigh number 10<sup>4</sup> is avoided because the Nusselt number can't be less than 1.

#### 4.2 Consequence of Rayleigh Number of different nano-fluid on heat transfer

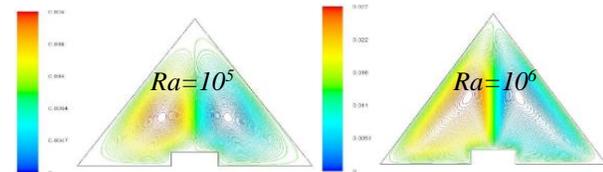
Contours of isotherm, stream function are shown below both for Al<sub>2</sub>O<sub>3</sub> and Cu nanofluid, for a constant time.



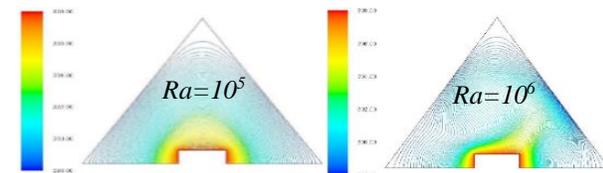
**Fig.11** Figure of Contour of Isotherm for various Rayleigh number (for Al<sub>2</sub>O<sub>3</sub>-  $\phi=0.1$ )



**Fig.12** Figure of Contour of Streamlines for various Rayleigh number (for Al<sub>2</sub>O<sub>3</sub>-  $\phi=0.1$ )



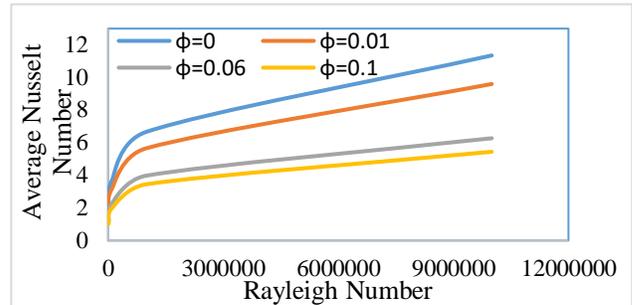
**Fig.13** Figure of Contour of Isotherm for various Rayleigh number (for Cu-  $\phi=0.1$ )



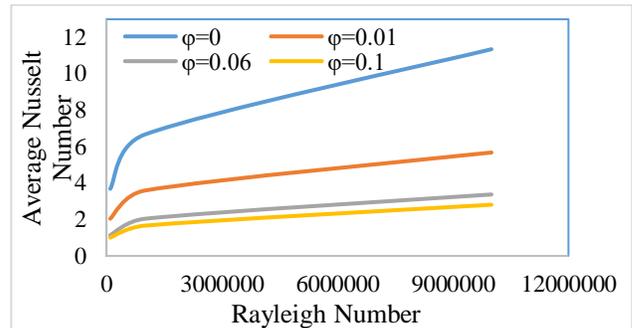
**Fig.14** Figure of Contour of Streamlines for various Rayleigh number (for Cu-  $\phi=0.1$ )

In the case of isotherm, when the value of Rayleigh number is small, there is no such change in contour, but when the value is increasing, there is a change visible in isotherm. Likewise, in the stream function, the stream function magnitude increases with the Rayleigh number. It takes a stable shape for a higher value of volume fraction. Because of the buoyancy effect, this change occurs.

A graph has been plotted between average Nusselt number with Rayleigh number for both Al<sub>2</sub>O<sub>3</sub>-water and Cu-water nanofluid for four different volume fractions. (Fig.15 and 16). It is seen that Nusselt number rises with increasing Rayleigh number and the value is decreasing with the volume fraction.

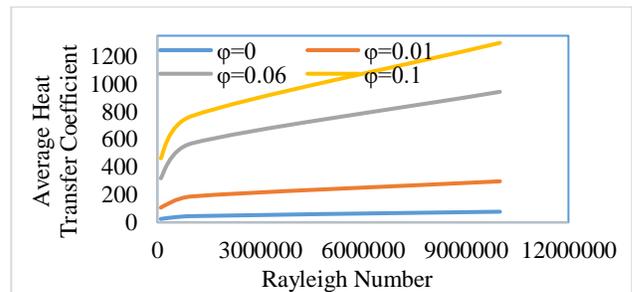


**Fig.15** Change of avg. Nusselt no. and Rayleigh no. (for Al<sub>2</sub>O<sub>3</sub>-  $\phi=0.1$ )

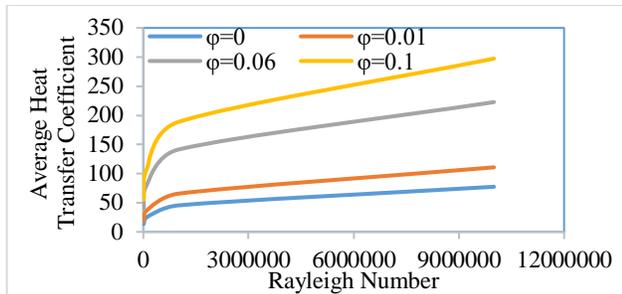


**Fig.16** Change of avg. Nusselt no. and Rayleigh no. (for Cu-  $\phi=0.1$ )

In Fig.17, it is shown that the average heat transfer coefficient increases in proportion to the Rayleigh number. Here it has been found that when the value of the volume fraction increases, the average heat transfer coefficient has also been increased. And it is found from (Fig.18) that the average heat transfer coefficients for Cu is greater than the values for Al<sub>2</sub>O<sub>3</sub> for the same volume fraction.



**Fig.17** Variation of average heat transfer coefficient with the Rayleigh number (for Al<sub>2</sub>O<sub>3</sub> -  $\phi=0.1$ )



**Fig.18** Variation of average heat transfer coefficient with the Rayleigh number (for Cu-  $\phi=0.1$ )

## 5. Conclusion

In this numerical investigation, several analyses have been carried out to enhance heat transfer process. The result of the analysis indicates that:

i) The convective heat transfer rate rises when the value of volume fraction increases.

ii) The Nusselt number decreases with the increment in volume fraction, but it reverses in case of Rayleigh number.

iii) Cu nanofluid is more effective than  $Al_2O_3$  for higher heat transfer.

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## NOMENCLATURE

$S$	: Height and Width of Cavity, m
$\phi$	: Fraction of nanoparticle volume
$T$	: Temperature, K
$P$	: Pressure, Pa
$t$	: Time, Second
$x, y$	: Dimensional Space Coordinate, m
$g$	: Gravitational acceleration, $ms^{-2}$
$Ra$	: Rayleigh number
$h$	: Heat transfer coefficient
$\alpha$	: Thermal diffusivity
$\beta$	: Thermal expansion coefficient of material( $K^{-1}$ )
$\mu$	: Dynamic viscosity, $Ns/m^2$
$\nu$	: Kinematic viscosity, $m^2s^{-1}$
$\rho$	: Density, $kg/m^3$
$k$	: Thermal conductivity of material, $W\cdot m^{-1}K^{-1}$
$C_p$	: Specific heat of material, $J\cdot kg^{-1}K^{-1}$
$Nu$	: Nusselt number
$nf$	: Nanofluid