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Numerical Analysis of Free Convection in a Lid-driven Cavity Having a Heated Circular Hollow Cylinder

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ABSTRACT

Free convection heat transfer introduced by the combined effect of mechanically driven lid and buoyancy force in a circular hollow cylinder. In the compound, the horizontal walls are maintained a uniform temperature at any point. The main focus will be the changes in pressure and velocity of our model with the changes in diameter and Ra & Gr no. The current study analyzed a practical system such as heat transfer in heated boiler cell. The governing equations for the problem are firstly converted into the non-dimensional form. The computation carried out with Grashof and Rayleigh number. The analysis is transmitted with the variation of streamlines and isothermals for different Rayleigh number ranging from 10^3 to 10^7 where Prandtl number kept 0.71 constant. Moreover, the results of this investigation are shown by the variation of fluid temperature in the enclosure with the different parameter (velocity, Pressure).

Keywords: Free Convection, Lid-driven Cavity, Grashof, Rayleigh Prandtl number

1. Introduction

Free convection investigation in lid-driven cavity is one of the most appurtenant to many environmental and engineering applications. It should be mentioned that the problem for viscous incompressible fluid flow in the lid-driven cavity is a well-known benchmark**.** The transfer of energy as heat is usually transfers to the lower temperature medium from the higher temperature one. But, when the two mediums reach the same temperature, heat transfer stops. Free-convective flows can be turbulent and laminar. A flow passed a solid surface with the temperature higher than that of the surrounding flowing medium, is the most extensive type of free convection. A lid driven cavity may create different pressure and velocity curves for different Parameters. The curves can be different for the difference of diameter of the cylinder hollow inside the lid driven cavity and other Parameters like some dimensionless numbers. With help of the simulation software '**COMSOL Multiphysics'** we get result and analyze of the research.

In cavities, a partition is assumed to enhance the heat transfer. We find many studies in an obstructed cavity on natural convection in the literatures. House et al [1] studied natural convection in a square vertical square including heat conducting objects. Dong associated with Li [2] conducted the compounded consequence of both the natural convection and conduction in an intricate compound. Numerical analysis of steady laminar of natural convection filled with a stable quantity of directed solid material consisting of either circular or square hindrance is conducted by Braga and Lemos [3]. On this investigation the researcher exhibited that for cylindrical rods, the average Nusselt number is moderately lower than those for square rods. Numerical investigation carried for the problem of laminar natural convective heat transfer in a square cavity with an adiabatic arc formed baffle by Tasnim and Collins [4]. Laskowski et al [5] exhibits both numerically and experimentally heat transfer to a circular cylinder in a cross-flow of water with low Reynolds number. The results described that, the temperatures of the lower surface and water upstream of the cylinder were carried on approximately equal and the flow was laminar when the lower surface was unheated, then Shih [6] investigated an intermittent laminar flow along with heat transfer due to an heat proofed or different constant temperature revolving objects like square, circular and the last is equilateral triangle settled in the middle of the cavity of square shape. Average Nusselt number of the respective systems of transient variations identified that for high Re numbers, a quasiperiodic behavior is established while for lower Re numbers, periodicity of the system is clearly identified .Sarkar et al [7] investigated.

2. Physical Model

In figure 1 it is sketched a physical system with the system of coordinates. In our problem, d is assigned as the diameter of hollow cylinder and 'k' is assigned as the thermal conductivity which is located at center of the square enclosure. The length of the square enclosure is 1m.

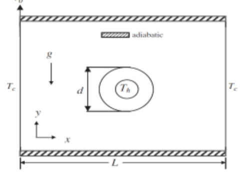

Fig.1 *Boundary condition diagram of the domain*

In our lid-driven cavity only the top wall can slide horizontally. We maintain the diameter of the inner circle d fixed at 0.05m but the outer diameter D is changed in 0.2m, 0.3m, 0.4m respectively. So the volumes of circular hollow cylinder were changed with the change of D.

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The two sidewalls maintain uniform constant temperature, *Tc* where the horizontal top and bottom walls are adiabatic. Moreover, the gravity performs in the downwards direction. Pressure, viscous dissipation radiation work are assumed to be omitted and it has been considered valid of Boussinesq approximation. All solid boundaries are considered to be rigid with no-slip walls.

3. Mathematical model Format:

The free convection differential governing equations express the mass conservation law, energy and last is momentum. The flow is assumed as Steady laminar flow along with incompressible and two-dimensional. With the exception of the buoyancy term the differences of the properties of fluid with temperature that is omitted, for the Boussinesq approximation has been adopted. The governing equations and the boundary conditions are in the dimensionless formation using the dimensionless variables.

In Y direction two-dimensional steady state problem with buoyancy forces acting, without internal heat generation, negligible viscous dissipation, dimensional governing equations are

Continuity equation $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

Hence, the X-momentum equation

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

Hence, the Y-momentum equation

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho g\beta\left(T-T_{ref}\right)$$

Energy equation $\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$

Here the parameters of the non-dimensional equations are,

Non dimensional coordinate, X = $\frac{x}{L_{ref}}=\frac{x}{L}$, Y $\frac{y}{l_{ref}}=\frac{y}{L}$

Non dimensional velocity, U$\frac{u}{u_{ref}}=\frac{ul}{\alpha}$, $V=\frac{u}{u_{ref}}=\frac{vl}{\alpha}$

Non dimensional pressure, $P=\frac{P}{P_{ref}}=\frac{Pl^2}{P\alpha^2}$

Non dimensional temperature, $\theta=\frac{T-T_{ref}}{\Delta T}=\frac{T-T_{\infty}}{\Delta T}$

Considering two-dimensional steady state problem with nobody forces, no internal heat generation and negligible viscous dissipation, non-dimensional governing equations are,

Continuity:- $\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}$=0

X-momentum:- U $\frac{\partial u}{\partial x}$ + V $\frac{\partial u}{\partial y}$ = - $\frac{\partial p}{\partial x}$ + $\frac{1}{R_{el}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)$

Y-momentum:- U $\frac{\partial v}{\partial x}$ + V $\frac{\partial v}{\partial y}$ = - $\frac{\partial P}{\partial Y}$ + $\frac{1}{R_{el}}\left(\frac{\partial^2 v}{\partial X^2}+\frac{\partial^2 v}{\partial Y^2}\right)$

Energy:- U $\frac{\partial\theta}{\partial X}$ + V $\frac{\partial\theta}{\partial Y}$ = $\frac{1}{R_{el}P_r}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right)$

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Using the definition:

$x=LX,\ y=LY,\ u=\frac{\alpha U}{L}$, $v=\frac{\alpha v}{L}$ in equation (i),

$$=>\frac{\partial\frac{\alpha u}{L}}{\partial XL}+\frac{\partial\left(\frac{\alpha v}{L}\right)}{\partial(LY)}$$

$$=>\frac{\alpha}{L^2}\left[\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right]=0$$

$$=>\frac{\partial V}{\partial X}+\frac{\partial V}{\partial Y}=0$$

This is the non-dimensional continuity equation.

Y-momentum equation:

$$P\left(U\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho_0 g\beta\left(T-T_{ref}\right)$$

…………………………………… (ii)

Using$x=LX,\ \ y=LY,\ u=\frac{\alpha U}{L},\ v=\frac{\alpha V}{L}$,

$P=\frac{\mu\alpha p}{L^2},\ \ T-T_{ref}\ =\Delta T\theta$ in equation (ii),

$$P_0\left[\frac{\alpha}{L}U\frac{\partial\left(\frac{\alpha V}{L}\right)}{\partial(LX)}+\frac{\alpha}{L}V\frac{\partial\frac{\partial V}{\partial(LY)}}{\partial(LY)}\right]=-\frac{\partial\left(\frac{\mu\alpha p}{L^2}\right)}{\partial(LY)}+\mu\left[\frac{\partial^2\left(\frac{\alpha V}{L}\right)}{\partial(LX)^2}+\frac{\partial^2\left(\frac{\alpha V}{L}\right)}{\partial(LY)^2}\right]+Pg\beta\Delta T\theta$$

$$P_0\frac{V^2}{L^3}\left[U\frac{\partial V}{\partial x}+v\frac{\partial V}{\partial y}\right]=-\frac{\mu V}{L^3}\frac{\partial p}{\partial y}+\mu\frac{V}{L^3}\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right]+Pg\beta\Delta T\theta$$

$$U\frac{\partial V}{\partial x}+v\frac{\partial V}{\partial y}=-\frac{\partial P}{\partial Y}+\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right]+\frac{g\beta L^3\Delta T\theta}{V^2}$$

$$U\frac{\partial V}{\partial x}+v\frac{\partial V}{\partial y}=-\frac{\partial P}{\partial Y}+\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right]+G_r\,.\theta$$

This is a non- dimensional Y-momentum equation.

4. Numerical Modeling:

Including mass, momentum and energy equations the nonlinear governing partial differential equations, are transferred into a system of integral equations by using the Galerkin weighted residual finite element method. Modified by imposition of boundary conditions the nonlinear algebraic equations are as obtained that are. Finally, for solving those linear equations triangular factorization method is applied.

Mesh Generation:

The mesh generation is the technique to subdivide a domain into a set of sub-domains, in finite element method called finite elements, control volume etc. By the numerical grid the discrete locations are defined.

The variables are calculated. By a collection of finite elements make the method a valuable practical tool for the solution of boundary value problems raising the various fields of engineering the computational domains with irregular geometries. Figure 2 shows the finite element mesh of the physical model.

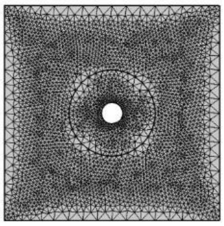
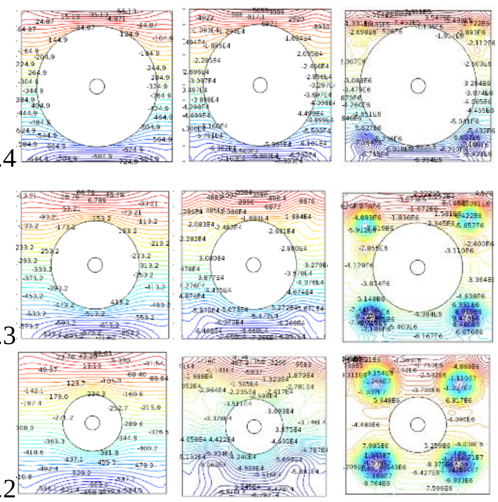


Fig.2 Mesh generated figure of free convection in square Lid-driven cavity

5. Model Simulation and result

5.1 Pressure Line:

Pressure lines, shown in figure 3, indicate the pressure level in the lid driven cavity generated by COMSOL Multiphysics. Different pressure levels are indicated by some colors. The red color indicates higher pressure and the yellow, green, sky, blue, refers lower pressure levels respectively.



D = 0.4

D = 0.3

D = 0.2

Ra $=10^3$ Ra$=10^5$ Ra$=10^7$

Fig.3 Pressure lines for different values of Ra and diameter.

The pressure lines seem similar to each other but they are not same to each other. The different pressure lines are created for the variation of the volume of heated circular hollow cylinder and the Rayleigh Number (Ra). We take the outer diameter (D) of the hollow cylinder at 0.4m, 0.3m and 0.2m where the inner diameter (d) is unchanged. The Rayleigh (Ra) numbers are taken by 10^3, 10^5,10^7.

When D = 0.4 and Ra = 10^3, the pressure decreases from upper portion to the lower portion and increases from left to right. It is noticed that an approximate similar difference is maintained in the pressure line. In the middle portion the pressure lines are maintaining some straight line whereas at the corner areas the pressure lines are tending to bend.

When D = 0.4 and Ra = 10^5, the pressure also decreases from upper portion to the lower portion and increases from left to right. An approximate similar difference also maintained in the pressure line. In the middle portion the pressure lines tend to be straight on the other hand at the corner areas the pressure lines seem to attempt making circular vortex.

When D = 0.4 and Ra = 10^7, the pressure decreases from upper portion to the lower portion and increases from left to right. An approximate similar difference also maintained in the pressure line. In the middle portion small vortexes are created but at the corner areas circular vortex created clearly.

When D = 0.3 and Ra = $10^{3,}$ the pressure decreases from upper portion to the lower portion and increases from left to right. It is noticed that an approximate similar difference is maintained in the pressure line. In the middle portion the pressure lines are maintaining some straight lines whereas at the corner areas the pressure lines tends to bend. It is also noticed that the pressure lines of lower corners are more banded than the pressure lines of upper corners.

When D = 0.3 and Ra =10^5, the pressure decreases also from upper portion to the lower portion and increases from left to right. It is noticed that an approximate similar difference is maintained in the pressure line. In the middle portion, the pressure lines tending to bend whereas at the corner areas the pressure lines are tend to make vortexes. It is also noticed that the pressure lines of lower corners are closer to create vortex than the pressure lines of upper corners.

When D = 0.3 and Ra = 10^7, the pressure lines do not maintain the color of pressure lines that maintained to others discussed before. In the vortexes created at upper corners we see the combination of different colors that indicate in the vortex making lines are not same in pressure. In the upper middle portion two small vortexes are created and in the lower middle portion pressure lines seem to create vortex. At the lower corners two vortexes are also created by different pressure lines.

When D = 0.2 and Ra = 10^3, the pressure decreases from upper portion to the lower portion and increases from left to right. It is noticed that an approximate similar difference is maintained in the pressure line. In the middle portion the pressure lines are maintaining some straight line whereas at the corner areas the pressure lines are tending to bend. It is also noticed that the pressure lines of lower corners are more banded than the pressure lines of upper corners. In this figure pressure lines are remarked with lower pressure with respect to the figure discussed for D= 0.2 and Ra=10^3

When D= 0.2 and Ra = 10^5, the pressure decreases also from upper portion to the lower portion and increases from left to right. It is noticed that an

Approximate similar difference is maintained in the pressure line. In the middle portion the pressure lines are tending to bend whereas at the corner areas the pressure lines are tend to make vortexes. It is also noticed that the pressure lines of lower corners are closer to create vortex than the pressure lines of upper corners. In this figure pressure lines are remarked with lower pressure with respect to the figure discussed for D= 0.2 and Ra=10^3

When D= 0.2 and Ra = 10^7, the pressure lines do not maintain the color of pressure lines that maintained to others discussed before. In the vortexes, created at upper corners, the combination of different colors that indicate the vortex making lines are not same in pressure. In the upper middle portion two small vortexes are created and in the lower middle portion pressure lines seem to create vortexes. At the lower corners two vortexes are also created by different pressure lines. In this figure pressure lines are remarked with lower pressure with respect to the figure discussed for D= 0.2 and Ra=10^3

5.2 Velocity Profile:

It has been seen that pressure lines velocity profile is used to indicate the velocity in the lid driven cavity generated by **COMSOL** shown in the figure 4, velocity points are indicated by some colors. The red color indicates higher velocity and the yellow, green, sky, blue, refers lower pressure levels respectively.

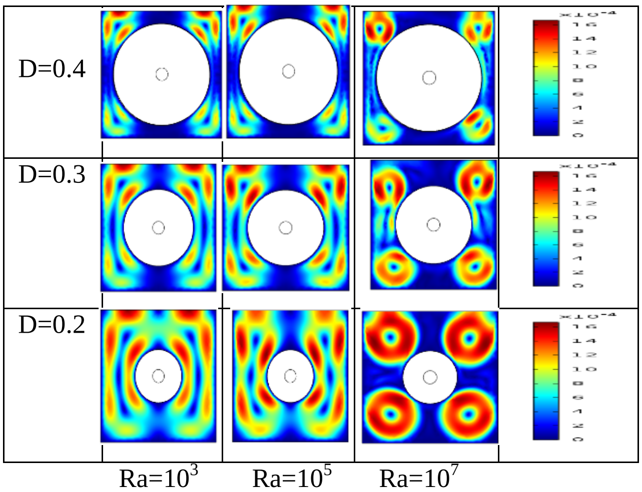


Fig.4 Velocity Profiles for different values of Ra and diameter.

The relation between pressure and velocity is indirectly proportional. The velocity profile can be explained with the help of pressure lines.

When D = 0.4 and Ra = 10^3, the velocity of upper portion is more than the lower portion and seems similar to left to right. The velocity seems same at left and right portion at same horizontal line. In the middle portion the velocity is zero or about to indicated by blue color. At the corners the velocity has some values.

When D = 0.4 and Ra = 10^5, the velocity of upper portion is also more than the lower portion and seems similar from left to right. The velocity seems same at left and right portion at same horizontal line. In the middle portion the velocity is zero or about to indicate by blue color. At the corners the velocity has some values. It seems at the corner vortexes are going to be formed.

When D = 0.4 and Ra = 10^7, vortexes are created at the corners. The velocity of upper vortexes is more than the lower velocity of lower vortexes. The velocity is not seems same at left and right portion at same horizontal line like before. In the middle portion the velocity is zero or about to zero indicated by blue color. At the Vortexes the velocity has some values.

When D = 0.3 and Ra = 10^3, the non-zero velocity of upper portion is more than the lower portion and seems similar from left to right. The velocity seems same at left and right portion at same horizontal line. In the middle portion the velocity is not less as we saw for D = 0.4 and Ra = 10^5. At the corners the velocities are more than when it was for D = 0.4 and Ra = 10^3

When D = 0.3 and Ra = 10^5, the velocity of upper portion is more than the lower portion and seems similar from left to right. The velocity seems same at left and right portion at same horizontal line. In the middle portion the velocity is zero or about to zero indicated by blue color. At the corners the velocity has some values.

When D = 0.3 and Ra = 10^7, vortexes are created at the corners. The velocity of upper vortexes is more than the lower velocity of lower vortexes. The velocity does not seem same at left and right portion at same horizontal line like before. The velocity at upper and lower middle area is very low. Sometimes it seems to be zero.

When D = 0.2 and Ra = 10^3, the non-zero velocity of upper portion is more than the lower portion and seems similar from left to right. The velocity seems same at left and right portion at same horizontal line. In the middle portion the velocity is lower. At the upper corners the velocities are more than the lower corners.

When D = 0.2 and Ra = 10^5, the velocity of upper portion is more than the lower portion and seems similar from left to right. The velocity seems same at left and right portion at same horizontal line. In the upper middle portion the velocity is about to zero. It is going to be formed four vortexes in the four corners.

When D = 0.2 and Ra = 10^7, vortexes are created at the corners. The velocity of upper vortexes is more than the lower velocity of lower vortexes. The velocity seems same at left and right portion at same horizontal line. In the middle portion the velocity is zero or about to indicated by blue color. At the vortexes are created with the velocity highest value. The center of the vortex is zero or about to zero.

6. Discussion

This numerical investigation shows that effects of magnetic field inside the moving (left and right) wall cavity. This analysis stipulate that the heat transfer and flow characteristics in this study stands on dimensionless five parameters which are the thermal conductivity ratio of solid fuel (K), diameter of the cylinder (D), Rayleigh number (Ra), Prandtl number (Pr), and Grashof number (Gr). The Numerical simulation results are presented in favor of velocity and pressure distributions for various values of Rayleigh number and different types of diameter of a hollow cylinder (heated) for free convection in a cavity (lid-driven). Numerical study is executed for different Rayleigh numbers $10^3 \leq Ra \leq 10^7$, where the value of Prandtl number (*Pr=0.71*), is fixed with changing Ra and the values of diameter are 0.2, 0.3 and 0.4 respectively. The lid driven cavity filled with fluid with a heated circular hollow cylinder which is located above the mid of the cavity with no slip boundary wall condition. The influence of Rayleigh number on the distribution of the velocity and pressure patterns is depicted in Model Simulation. Symmetrical flow heat transfer and this effect is more pronounced with increasing the values of Rayleigh Number. As the value of the Rayleigh number increases, multicellular structure is observed within the cavity for lower diameter cylinder. Flowing fluid and transfer of heat are strongly influenced by the radius/diameter of the hollow shaped cylinder. This simulation finds that hollow cylindrical shape has an important impact to the outflow in the total hybrid area of the cavity.

7. Conclusion

Current investigation in a lid-driven cavity having a circular hollow cylinder addresses numerical study on two-dimensional laminar free convection that has been analyzed numerically for the various governing parameters like the solid fluid thermal conductivity ratio K, a cylinder diameter D, and Raleigh number Ra. Fluid that flows inside a cavity of our model and a heat transfer attributes inside the cavity that is not depending on the fluid thermal conductivity (Solid) ratio (K) for the free type convections. On the distribution of fluid locomotion and temperature in terms of velocity and temperature profiles the effects of the mentioned parameters were used. For pure mixing and vortex formulation the diameter of the cylinder and the Rayleigh number has very strong influence.

8. References

[1] J.M. House, C. Beckermann, T.F. Smith, Effect of a centered conducting body on natural convection heat transfer in an enclosure, Numer, Heat Transf., A 18 (1990) 213–225.

[2] S.F. Dong, Y.T. Li, Conjugate of natural convection and conduction in a complicated enclosure, Int. J. Heat Mass Transf. 47 (2004) 2233–2239

[3] E.J. Braga, M.J.S. de Lemos, Laminar natural convection in cavities filed with circular and square rods, Int. Commun. Heat Mass Transf. 32 (2005) 1289–1297.

[4] S.H. Tasnim, M.R. Collins, Suppressing natural convection in a differentially heated square cavity with an arc shaped baffle, Int. Commun. Heat Mass Transf. 32 (2005) 94–106

[5] G. Laskowski, S. Kearney, G. Evans, R. Greif, Mixed convection heat transfer to and from a horizontal cylinder in cross-flow with heating from below, Int. J. Heat Fluid Flow 28 (2007) 454–468

[6] Y. Shih, J. Khodadadi, K. Weng, A. Ahmed, Periodic fluid flow and heat transfer in a square cavity due to an insulated or isothermal rotating cylinder, J. Heat Transf. 131 (2009) 1–11

[7] Sarkar, M.A. Alim, M.A. Munshi, M. J. H., & Ali, M. M. (2019). Numerical study on MHD mixed convection in a lid driven cavity with a wavy top wall and rectangular heaters at the bottom. 8th bsme international conference on thermal engineering. doi:10.1063/1.5115849

NOMENCLATURE

ρ	Density, kg/m3
T_h	Hot wall
T_c	Cold wall
k	thermal conductivity
Cp	Specific heat at constant pressure, kJ/kg. K
Pr	Prandtl number
F_y	(Ra/Pr)
G	Acceleration due to gravity , m/s^2
Gr	Grashof Number
Ra	Rayleigh Number
T	Fluid temperature
T_∞	Temperature of ambient fluid; cold wall
U	Velocity component in x directional axis
V	Velocity component in y directional axis
X	Co-ordinate parallel to the plate
Y	Co-ordinate normal to the plate
μ	Dynamic viscosity
L_{ref}	Length (reference)
U_{ref}	Velocity (reference) $\left(\frac{a}{l}\right)$
P_{ref}	Reference pressure, $\frac{\rho\alpha\mu}{L_{ref}}$
T_{ref}	Reference temp., T_∞
A	Heat transfer area [m2]
Q	Conduction heat transfer [W]
∂_T / ∂_x	Temperature gradient [Km-1]
T_s	Temperature(abs) of the hot surface [K]
T_∞	Temperature(Abs) of the cold surface [K]
$T_s - T_\infty$	Temperature difference [K]
h	Convective heat transfer coefficient, $W/m^2.K$
ε	Surface emissivity