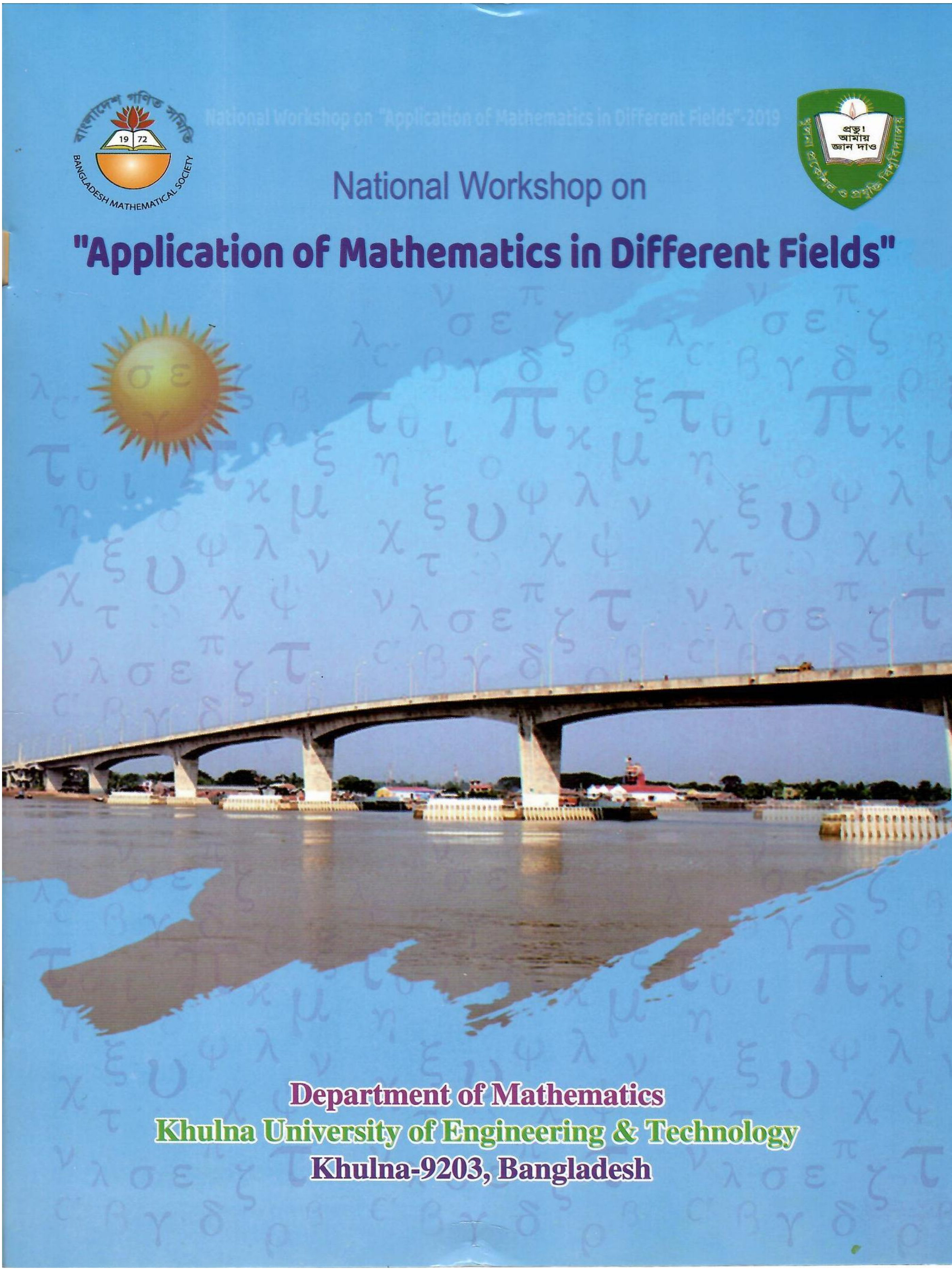

BANGLADESH MATHEMATICAL SOCIETY
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National Workshop on "Application of Mathematics in Different Fields"-2019
National Workshop on
"Application of Mathematics in Different Fields"
Department of Mathematics
Khulna University of Engineering & Technology
Khulna-9203, Bangladesh

National Workshop on *"Application of Mathematics in Different Fields"*-2019

Program Schedule:
Date: 20 December, 2019

Session-I: Inaugural Session

Time	Purpose	Venue
08:00 am-09:00 am	Registration and Kit distribution	Ground Floor Lobby Students Welfare Center, KUET
09:00 am-09:30 am	Opening Ceremony	Room No. 201 Students Welfare Center, KUET
09:30am-10:00 am	Tea Break	1st Floor Lobby Students Welfare Center, KUET

Session-II: Technical Session

Time	Purpose	Venue
10:00 am-10:45 am	Application of Calculus, Differential equation and Fourier series: Prof. Dr. B. M. Ikramul Haque	Room No. 201 Students Welfare Center, KUET
10:45 am-11:30 am	Mathematics in Finance: Dr. A. B. M. Shahadat Hossain	
11:30 am -12:15 pm	Fractional Differential Equations: Prof. Dr. M. Ali Akbar	
12:15 pm -01:00 pm	Computational Fluid Dynamics: Prof. Dr. Md. Ashraf Uddin	
01:00 pm -02:30 pm	Prayer and Lunch Break	
02:30 pm -03:15 pm	Application of Matrices: Prof. Dr. A. R. M. Jalal Uddin Jamali	Room No. 201 Students Welfare Center, KUET
03:15 pm -04:00 pm	Fluid Dynamics: Prof. Dr. Md. Anwar Hossain	
04:00 pm-04:30 pm	Certificate giving and closing ceremony	

National Workshop on

“Application of Mathematics in Different Fields”

Organized by:

Department of Mathematics
Khulna University of Engineering & Technology
Khulna-9203, Bangladesh

In collaboration with
Bangladesh Mathematical Society

December 20, 2019

Program Coordinator:

Dr. Md. Bazlar Rahman, Professor, Department of Mathematics, KUET

Program Secretary:

Dr. A. R. M. Jalal Uddin Jamal, Professor, Department of Mathematics, KUET

Disclaimer:

Organizing Committee and Bangladesh Mathematical Society are not responsible for any error or view expressed in the write-ups as these are the responsibilities of the individual author.

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Message

I am really delighted to know that the Department of Mathematics, Khulna University of Engineering & Technology (KUET) is going to organize a National Workshop on **"Application of Mathematics in Different Fields"** for the first time in collaboration with Bangladesh Mathematical Society (BMS).

From the very outset of our formal education we have experienced some applications of arithmetic and algebra related to our daily lives. In this era of technological advancement, Mathematics appears to be crucial in all fields of knowledge: Science, Engineering, Business, even Social Science. So we can no longer avoid learning mathematics from the application point of view. Therefore, I firmly believe that this workshop will add value to the knowledge of the participants as it will explore the applicability of mathematics to various fields of higher education as well as it will provide them with the opportunity to know more about the applied side of mathematics in their own research domain and in other fields as well.

I hereby welcome all the participants, distinguished guests and all the resource persons in our green campus to make the program successful.

I must thank and express my heartfelt gratitude to BMS for supporting the Department of Mathematics to organize the National Workshop on **"Application of Mathematics in Different Fields"** and hope that BMS will arrange such scholarly workshop more often. I also hope that the Department of Mathematics, KUET will do their best to uphold the dignity as the accomplished organizer.

I wish a grand success of this workshop.

(Professor Dr. Quazi Sazzad Hossain)
Vice-Chancellor
Khulna University of Engineering & Technology
Khulna-9203, Bangladesh.

Message

The information that the Department of Mathematics of Khulna University of Engineering & Technology (KUET) is going to arrange the National Workshop titled "**Application of Mathematics in Different Fields**" to be held on 20th December, 2019 has delighted me very much. Mathematics has been devised of years back and that was due to its applicability, that means mathematics was started its journey to be applied for the betterment of human race. Even though up to our school level courses its applications are stated pages after pages at all the classes. From HSC level to onwards the mathematics courses are mainly theoretical, especially except a very few courses at bachelor level there are lots of theorems and problems in different branches of mathematics courses. Students of this level are thinking, also ourselves were done that, mathematics is a process to solve the theoretical problems. But the very title of this workshop expresses that mathematics in not only has its applications in its own field but also has applications in other fields.

So far I know, the intention to arrange such a workshop is to provide a very few ideas about the applicability of mathematics in different fields, especially to the teachers, so that they can put forward some examples to their students to make them curious about the subject.

Lastly, I want to thank the Department of Mathematics of KUET to think in that light and exploring the opportunity to arrange such a valuable workshop. I want to thank also the Bangladesh Mathematical Society to support the idea of the Department and to the various organizations for extending their hands to bring the dream of the Department in reality. As being the Dean of the Faculty of Civil Engineering, in which the Department of Mathematics belongs, it is my duty to support this type of events and so I have extended my support to make this event a successful one. I hope that the Department of Mathematics of KUET will try to arrange this type workshops on a regular basis.

M Arif Hossain

(Professor Dr. Mohammad Arif Hossain)
Dean
Faculty of Civil Engineering
Khulna University of Engineering & Technology
Khulna-9203, Bangladesh.

Message

Greetings!

As Head of the Department of Mathematics, Khulna University of Engineering & Technology (KUET), I take great pride in welcoming all the attendees of the National Workshop on "**Application of Mathematics in Different Fields**". Undoubtedly mathematics is one of the most common and core subjects to the academicians who are involved especially in teaching science, engineering, and technological subjects all over the world and hence play a vital role in producing skilled human resources by disseminating the knowledge of Mathematics.

At present mathematics is in extensive use not only in the stated disciplines but also in medical science, business and social science disciplines. You know the newly coined term "**Data Science/Scientist**" where mathematics is essential and it is evident that all multinational companies are employing "Data Scientist" with comparatively high salary in their organizations to make their firms more functional and profitable. Any person with a degree in mathematics having sufficient knowledge in applied mathematics can easily build his/her carrier in **Data Science**. It is just an example of modern job fields where mathematicians are highly welcomed. I hope this workshop will open the new window for the participants through which every attendee will certainly have some ideas where mathematics can be used efficiently. I hope this will play as a stimulating factor for the participants igniting them to discover the areas where application of mathematics will add value to the corresponding fields.

I thank Bangladesh Mathematical Society for supporting us to arrange such an outstanding National Workshop for the first time in Bangladesh. I do also thank all the resource persons for their valuable knowledge sharing and express my gratitude to University Grant Commission of Bangladesh as well as other benefactors for their great financial contribution. Moreover, I thank all committee members because of their hardworking to present us an excellent workshop and all the participants as you really into mathematics.

Finally, I strongly believe that Department of Mathematics, KUET have tried their level best to make the workshop convenient for you and successful as well. Despite all of our endeavors to make the program smooth and hassle free, we may have some shortcomings. I request all of you consider the issue generously if any inconvenience happened to you.

I wish this workshop will enlighten all of us in applying mathematics in various fields.

(Professor Dr. Md. Zaidur Rahman)
Head, Department of Mathematics
Khulna University of Engineering & Technology
Khulna-9203, Bangladesh.

Message from President of BMS

I have the pleasure to know that the Department of Mathematics at Khulna University of Engineering & Technology (KUET) is going to organize a day long workshop entitled "**Application of Mathematics in Different Fields**" on 20th December, 2019 in collaboration with Bangladesh Mathematical Society (BMS). BMS has a subcommittee to organize seminar, symposium, workshop, etc. round the year with the active participation of different universities, research institutes and colleges in home to illuminate mathematics education and research among young mathematician as well as prospective research students. The current workshop will be worthy not only for the researchers and educators at KUET but also for the other participants. I hope a galaxy of resource persons who are well versed in their respective fields will deliver very enthusiastic lectures which will help the participants of this workshop achieving their aims and aspirations.

On behalf of Bangladesh Mathematical Society I wish the event a great success.

(Professor Dr. Md. Mubarak Hassain)
President
Bangladesh Mathematical Society.

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Application of Mathematics

Professor Harunur Rashid

Mathematics is the oldest of all sciences and it has been applied for the development of all other sciences. It is a common belief that mathematics is used only in the science of motion and engineering. Let us start from the beginning of the day, i.e., the hour of sun rise. Mathematics has been applied to determine the hours of sun rise and sun set, the times of tied (high tide and ebb tide), the day of new moon and full moon, the months and year of Bangla year, the Christian year and the Arabic (Hijri) year, the hours and days of prayer of people of different religions. A bicycle rider, an automobile driver, a rickshaw puller, a cart puller, the captain of ship in the sea, the pilot of an aeroplane and similar such persons have to apply mathematics more or less for their safe journey. Mathematics has given rhythm to poems, melody in music, specially in classical music and a very attractive recreation in dance.

Mathematics was applied to determine the number and position of the planets and other solar bodies. The days of solar eclipse and lunar eclipse are also determined with the application of mathematics. It is needless to say that mathematics plays the most vital role in all engineering fields. Apart from various academic subjects, mathematics has very successfully applied in all outdoor games. The application of mathematics can help to have a possible shopping and a salesman to have a good sale. Mathematics has been applied to Banking, Insurance and Probability. Forecasting the weather as well as the attack of natural calamity like cyclone, flood and unnatural rise or fall of daily atmospheric temperature and pressure are all due to application of mathematics.

It is probably only in our country that study of mathematics is not required for those who want to study medicine and surgery. This is a serious miss-information. It is our common experience that a doctor enquires about the age, weight, height etc. of his patient before preparing his prescription. Those information help the doctor to determine the dosage, interval and length of the period for which the medicine has to be taken. Doctors who don't have aptitude of mathematical calculations can not prepare a balanced prescription. In order to determine the BMI (Body Mass Index) a bit of mathematical calculation is required. This can be done by every individual by using a simple mathematical formula (weight in pounds) × (constant) 704.7/(height in inches)^2 .
A normal BMI < 25 indicates good health, a BMI between 25 and 29.9 is considered to be overweight and a BMI > 30 is alarming overweight and demands care and treatment. Medical professionals use mathematics when preparing statistical graphs of epidemics or of success rates. Mathematics is applied to X-ray and CAT scans. The modern pathology that is used for diagnostic purpose entirely depends on numbers and numbers in percentage. Cardiac surgeon performs medical procedures on patients with heart problems. These surgeons monitor their patients on a daily basis before and after medical procedures to prevent complications.

To our surprise, the application of mathematics has made it easier to represent our spherical three dimensions planet 'Earth' on a flat two dimensional map. The application of mathematics in the invention of computer is a milestone in the development of civilization.

It is mathematics which was applied a few centuries ago for the discovery of various planets and solar bodies; it is again, mathematics which is applied to the science of space travel that carried man from the surface of the Earth to the surface of the Moon and the planet Mars.

Mathematics in nature: In the year 1202, Leonardo of Pisa explained an arithmetical sequence 1, 2, 3, 5, 8, 13, 21 etc. The formula for calculating these numbers is $F_n = F_{n-1} + F_{n-2}$.
These numbers are used to determine the Golden Ratio 1:1.618, which appears naturally in all aspects of life, petals on a rose, body ratio in human body. It is said that people that are considered naturally beautiful have a face and body that follows the golden ratio.

The Fibonacci numbers are very well represented in honeybees. By following the family tree of honeybees, it can be observed that it satisfies the Fibonacci sequence perfectly.

Number of	Parents	Grand Parents	Great grand parents	Great great grand parents	Great great great grand parents
Male Bess	1	2	3	5	8
Female Bees	2	3	5	8	13

By dividing the number of female bees the number of male bees, the golden ratio 1.618 is obtained. The mathematical sequence works for any honeybee live at any given time. Commonly honeybee lives are always used to explain the Fibonacci sequence and golden ratio.
Source: Internet

Application of Matrix: Latin Hypercube Experimental Design

A R M Jalal Uddin Jamali
Professor, Department of Mathematics
Khulna University of Engineering & Technology, Khulna-9203.

In mathematics, a matrix is a rectangular array of numbers, symbols, or expressions, arranged in *rows* and *columns* provided that they have the same size. That is if a matrix A has m rows and n columns then the matrix has nm entities and it is denoted as $m \times n$ (m by n) matrix. For example **X** is a $N \times k$ matrix:

$$\mathrm{X} = \begin{bmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nk} \end{bmatrix}$$

According to the above definition, matrix is just a well defined representation of entries which may be numbers, symbols, or expressions etc. But one of the main powers of matrix is hidden in its algebraic operation. Finally, its super power is shown when it acts as a linear operator. It has active touch almost all fields in which the term science is tagged such as Basic Science, Engineering Science, Medical Science, Business Science, Social Science etc. Here I would like to discuss briefly about a typical application of matrix on Design of Experiment (DoE) which is very impotent tool in the modern Science and Technology. The output of DoE is used as an input of surrogate model in computer simulations.

Physical experiments are often inevitabily very expensive and time consuming. So computer experiments are frequently used for simulating physical characteristics. But the computer simulation of mathematical model of the physical experiments is also usually time-consuming and there is a great variety of possible input combinations. So surrogate model (also called a global/ approximation model or meta-model) are required. Surrogate model is constructed based on modeling the response of the simulator to a limited number of cleverly chosen data points. It models the quality characteristics as explicit functions of the design parameters. The process comprises of three major steps which may be interleaved iteratively:
i) Sample selection (**DoE** or active learning).
ii) Construction of the surrogate model and optimizing the model parameters (Bias-Variance trade-off) .
iii) Appraisal of the accuracy of the surrogate.

Among the three steps of an experiment, the first and most important one is the selection of sample data. The very terminology DoE simply means to select cleverly a limited number of design points which are space-filling over the design space. It is known that the selected design points are used in surrogate model of the physical problem to understand the behavior of input variables on response surface. Moreover, after increasing the power of computer, the researchers are interested on simulations rather than actual experiments which are expensive and also time consuming. But simulation model of the physical problem is also time-consuming as it has huge variety of input combination. Therefore, DoE is very important to find out the limited number of sample points for the simulation model. The DoE is the procedure to find out the sample points which should have some impotent properties and will be discussed later.

The DoE which is used in simulation has much recent interest and this is likely to grow as more and more. It is worthwhile to mention here that many simulation models involve several hundred factors or even more. Computer simulation experiments are used in a wide range of application to learn about the effect of input variables $\boldsymbol{x}$ on, a response of interest y. The main objective of DoEs is to find out sample of a limited number of cleverly chosen data points such that sample data points are spread all over the domain space of experiments. Moreover as the effect of each factor on response surface is priori unknown so DoEs should have non-collapsing property. Actually a good DoE should have three important properties namely (i) space-filling (ii) non-collapsing and (iii) orthogonal. Space-filling property indicates the sample data points are spread over the domain so that these limited number of data points are able to collect information from all over the experimental domain. It is noted that when no details on the functional behavior of the response parameters are available, it is important to be able to obtain information from the **entire** design space. Non-collapsing property ensures that if design points are projected on each parameter axis then all the points are evenly spread over the axis. When DoE does not satisfy this property then, if one of the design parameters has (almost) no influence on the function value, two design points that differ only in this parameter will collapse, i.e., they can be considered as the same point that is evaluated twice. The third important property is that the factors of design points are as much as possible non-correlated which is called orthogonal property. It is worthwhile to mention here that if one or more factors are correlated than the individual effect on response surface are impossible to find out. But it is almost impossible to find a DoE which satisfies all the three properties smaltenusly. Among the several

experimental designs, Latin Hypercube Design (LHD) is one of the most chosen DoE in the field of application. It has a nice and unique characteristic which is called non-collapsing property. It is noted that Latin Square design, which is popular in physical experimental domain, is a special LHD where number of factors are 2.
But what is the relation between Matrix and LHD? Actually a LHD is represented by a matrix as given below. Consider a set of N design points in a uniform k-dimensional grid $\{0,1,2,\ldots,N-1\}^k$. A configuration

$$\mathrm{X} = \begin{bmatrix} \mathrm{X}_1 \\ \vdots \\ \mathrm{X}_N \end{bmatrix} = \begin{bmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nk} \end{bmatrix} \forall x_{ij} \in \{0,1,2,\ldots,N-1\}$$

Then **X** be a LHD iff $x_{ip} \neq x_{iq} \exists p \neq q$. i.e. each column has no duplicate entries.

To illustrate the LHD, we consider a typical LHD **X** which has 8 design points and two factors i.e.(N, k) = (8, 2).

It is worthwhile to mention here that the number of possible LHDs of (N, k) is so huge namely $(N!)^k$.

Anyway, let us consider the X as follows:

$$\mathrm{X} = \begin{bmatrix} \mathrm{X}_1 \\ \mathrm{X}_2 \\ \mathrm{X}_3 \\ \mathrm{X}_4 \\ \mathrm{X}_5 \\ \mathrm{X}_6 \\ \mathrm{X}_7 \\ \mathrm{X}_8 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 2 & 6 \\ 3 & 1 \\ 4 & 4 \\ 5 & 7 \\ 6 & 2 \\ 7 & 5 \end{bmatrix}$$

Let us consider another DoE **Y** of (8,2) given below:

$$\mathrm{Y} = \begin{bmatrix} \mathrm{Y}_1 \\ \mathrm{Y}_2 \\ \mathrm{Y}_3 \\ \mathrm{Y}_4 \\ \mathrm{Y}_5 \\ \mathrm{Y}_6 \\ \mathrm{Y}_7 \\ \mathrm{Y}_8 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 2 & 6 \\ 3 & 1 \\ 1 & 4 \\ 5 & 7 \\ 6 & 3 \\ 7 & 5 \end{bmatrix}$$

The pictorial view of LHD (1) and another DoE (2) are shown in Figure 1(a) and 1(b) respectively. It is remarked that though both are DoE for (N, k) = (8, 2) but 1(a) is LHD and 1 (b) is not LHD. But why?

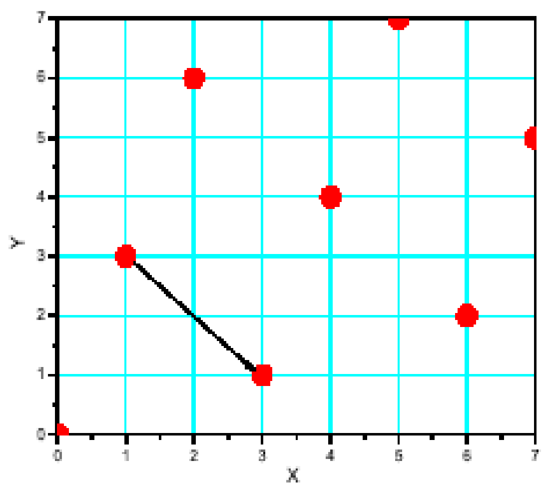

Fig. 1(a): LHD for (N, k) = (8, 2)

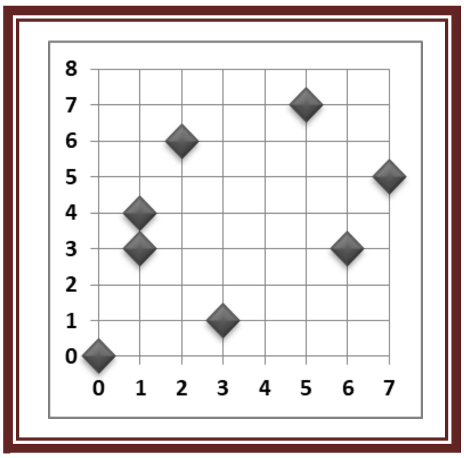

Fig. 1(b): DoE but not LHD for (N, k) = (8, 2)

If we will project the design point of LHD X on first coordinate x then which is done by matrix transform as follow:

$$\operatorname{Proj}(X)\,\text{on } x \text{ axis} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 2 & 6 \\ 3 & 1 \\ 4 & 4 \\ 5 & 7 \\ 6 & 2 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{bmatrix}$$

i.e. we have $\{0,1,2,3,4,5,6,7\}$ which are evenly spread over the x axis and no any point overlap on the axis. Again if we project the design point of LHD X on second coordinate y then we have as follows:

$$\operatorname{Proj}(X)\,\text{on } y \text{ axis} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 2 & 6 \\ 3 & 1 \\ 4 & 4 \\ 5 & 7 \\ 6 & 2 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 6 \\ 1 \\ 4 \\ 7 \\ 2 \\ 5 \end{bmatrix}$$

i.e. we have $\{0,1,2,3,4,5,6,7\}$ which are evenly spread over the y axis and no any point overlap on the axis. On the other hand if we project the design point of DoE **Y** on second coordinate y then we have

$$\operatorname{Proj}(Y)\,\text{on } y \text{ axis} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 2 & 6 \\ 3 & 1 \\ 1 & 4 \\ 5 & 7 \\ 6 & 3 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 6 \\ 1 \\ 4 \\ 7 \\ 3 \\ 5 \end{bmatrix}$$

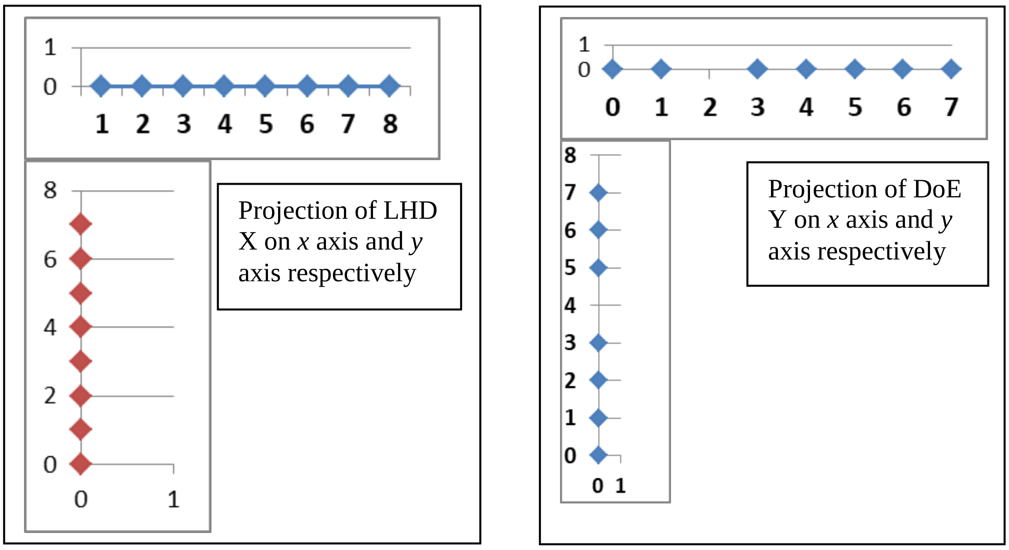

Figure 2: Projection of LHD X and DoE Y on both axes

{0, 1, 2, **3**, , 5, 6, 7} which are not evenly spread over the axis y and point **3** is overlapped on the axis. Similarly, if we project the design point of DoE **Y** on second coordinate y then we have {0, **1**, , 3, 4, 5, 6, 7} which are not evenly spread over the axis and point **1** is overlapped on the axis. The pictorial views are shown in Figure 2.

It is noted that in simulation model primarily DoE considers some more factors. But it is often observed that only few factors have significant effect on response surface. The LHD preserve it all properties after removal insignificants factor if any. But this operation is also done by matrix transformation. For instance we consider a LHD Z for 6 design points with 5 factors which is given below:

$$\begin{bmatrix} 0 & 4 & 4 & 4 & 4 \\ 1 & 0 & 2 & 0 & 3 \\ 2 & 5 & 1 & 1 & 1 \\ 3 & 1 & 0 & 5 & 2 \\ 4 & 2 & 5 & 3 & 0 \\ 5 & 3 & 3 & 2 & 5 \end{bmatrix}$$

Now suppose the second factor (i.e. second column) has no influence on response surface. So we need a LHD with reduces dimension namely 4 dimension rather than 5 in which the second factor will be removed. We can again do this successfully by using matrix operation as follow:

$$\begin{bmatrix} 0 & 4 & 4 & 4 & 4 \\ 1 & 0 & 2 & 0 & 3 \\ 2 & 5 & 1 & 1 & 1 \\ 3 & 1 & 0 & 5 & 2 \\ 4 & 2 & 5 & 3 & 0 \\ 5 & 3 & 3 & 2 & 5 \end{bmatrix} . \begin{bmatrix} 1 & \mathbf{0} & 0 & 0 & 0 \\ 0 & \mathbf{0} & 0 & 0 & 0 \\ 0 & \mathbf{0} & 1 & 0 & 0 \\ 0 & \mathbf{0} & 0 & 1 & 0 \\ 0 & \mathbf{0} & 0 & 0 & 1 \end{bmatrix}$$

After multiplication we have

$$\begin{bmatrix} 0 & 0 & 4 & 4 & 4 \\ 1 & 0 & 2 & 0 & 3 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 5 & 2 \\ 4 & 0 & 5 & 3 & 0 \\ 5 & 0 & 3 & 2 & 5 \end{bmatrix}$$

Now we interchange by elementary column transformation namely $C_{1,2}$ we have

$$\begin{bmatrix} 0 & 0 & 4 & 4 & 4 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 3 & 0 & 5 & 2 \\ 0 & 4 & 5 & 3 & 0 \\ 0 & 5 & 3 & 2 & 5 \end{bmatrix}$$

Finally, we have to remove the fist column. We can again do this by matrix transformation as follow:

$$\begin{bmatrix} 0 & 0 & 4 & 4 & 4 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 3 & 0 & 5 & 2 \\ 0 & 4 & 5 & 3 & 0 \\ 0 & 5 & 3 & 2 & 5 \end{bmatrix} . \begin{bmatrix} \mathbf{0} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i.e. we have the following DoE which is off course also LHD.

$$\begin{bmatrix} 0 & 4 & 4 & 4 \\ 1 & 2 & 0 & 3 \\ 2 & 1 & 1 & 1 \\ 3 & 0 & 5 & 2 \\ 4 & 5 & 3 & 0 \\ 5 & 3 & 2 & 5 \end{bmatrix}$$

Matrix has emerging applications in the fields of computer science and operation research. Another important application of matrix is eigenvalue problem. When you will able to represent the mathematical model of any physical model as matrix representation then you can able to analyze the systems intensively. The mathematical model may be the systems of algebraic or differential equations. Actually almost all linear systems can be model as matrix and you can solve the problem more easily.

A Brief Introduction to Certain Applications of Differential Equation and Necessity of its Solution

Dr. B. M. Ikramul Haque
Professor, Department of Mathematics
Khulna University of Engineering & Technology, Khulna-9203.

The subject of differential equation not only is one of the most beautiful parts of mathematics, but it is also an essential tool for modeling many physical situations such as spring mass system, resistor-capacitor-inductor circuits, bending of beams, chemical reactions, pendulums, the motion of the rotating mass around another body and so forth. The differential equations may be linear or nonlinear, autonomous or non-autonomous. Practically, numerous differential equations involving physical phenomena are nonlinear. In many cases it is possible to replace such a nonlinear equation by a related linear equation, which approximates the actual nonlinear equation closely enough to give useful results. However, such a “linearization” is not always feasible; and when it is not, the original nonlinear equation itself must be considered.

In this article I shall give a brief introduction to certain applications of linear and nonlinear differential equation and some solution methods for nonlinear differential equations.

I. Some Applications of First Order Differential Equation:

First order linear differential equations are used very frequently in solving problems related to electrical circuits, radioactive decays, carbon dating, population dynamics, mixture problems, Newton cooling *etc*.

Law of decay: If the rate of change of a quantity m at any instant t is proportional to the quantity present at that time then the differential equation of decay is $\dfrac{dm}{dt} = -k\,m$

Law of growth: If the rate of change of a quantity m at any instant t is proportional to the quantity present at that time then the differential equation of growth is $\dfrac{dm}{dt} = k\,m$

Example 1: **(Newton’s Law of cooling)**

It states that the rate of change temperature of a body is directly proportional to the difference between the temperature of the body and the temperature of the surrounding medium.

Modeling for cooling:

If θ be the temperature of the body and θ_0 be the temperature of the surrounding medium, then

$$\frac{d\theta}{dt} = -k\,(\theta - \theta_0)$$

Modeling for heating:

If θ be the temperature of the body and θ_0 be the temperature of the surrounding medium, then

$$\frac{d\theta}{dt} = k\,(\theta - \theta_0)$$

Net rate of change:

If be the body is cooling at the rate $k\theta$ heated at rate αt then the net rate of change of temperature is

$$\frac{d\theta}{dt} = -k\,\theta + \alpha t$$

II. Application of Second Order Ordinary Differential Equation

Example 2: A sky diver of mass m falls long enough without a parachute that is the drag force has strength kv^2 to reach his first terminal velocity v_1. When his parachute opens, the air resistance force has strength kv. At what minimum altitude must his parachute open so that he slows to within 1% of his new (much lower) terminal velocity v_2 by the time he hits the ground?

Let y denote the vertical distance measured downward form the point at which his parachute opens (which will be designated time $t = 0$). Then Newton's Second Law ($F_{net} = ma$) becomes $mg - Kv = ma$, or, since $v = \frac{dy}{dt}$ and $a = \frac{d^2y}{dt^2}$. Then $mg - K\frac{dy}{dt} = m\frac{d^2y}{dt^2}$

This situation is therefore described by the IVP

$$\frac{d^2y}{dt^2} + \frac{K}{y}\frac{dy}{dt} = g \text{ with } y(0) = 0, \quad \left.\frac{dy}{dt}\right|_{t=0} = v_1$$

The differential equation is second order linear with constant coefficients.

III. Application of Second Order Non-linear Differential Equation

First of all, a number of examples have been provided from various scientific fields that give rise to nonlinear equations.

Pendulum

One of the simplest nonlinear oscillating systems is the free pendulum. This system consists of a particle of mass m attached to the end of a light inextensible rod, with the motion taking place in a vertical plane. As shown in Figure, let θ be the angle between the vertical line and the line *OP*, where *O* is the center of the circular path and *P* is the instantaneous position of the particle. The distance S is measured from the equilibrium position O_1. From the figure 1 it follows that the component of the force of gravity mg in the direction of S is equal to — mg sin θ. If L is the length of the pendulum, then the equation of the pendulum is

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$

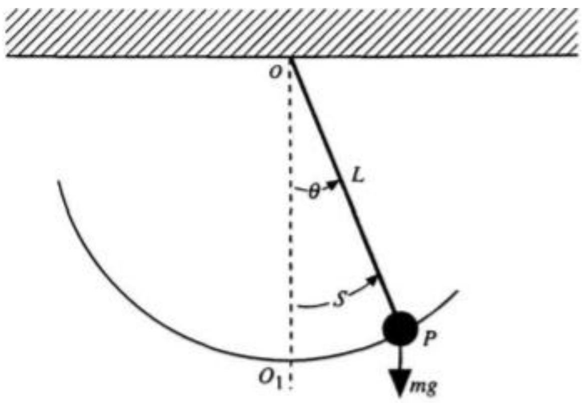

Fig-1

Mass Attached to a Stretched Wire

Consider the motion of a particle of mass *m* attached to the center of a stretched elastic wire. Let the ends of the wire be fixed a distance 2*d* apart, as shown in Figure 2. Assume that the particle is constrained to move only in the horizontal or *x* direction. If Hooke's law holds for each portion of the stretched wire, then the tension *T* in each part of the wire is $T = k(L - a)$, where *L* is the stretched length, *a* is the length for which *x = 0*, and *k* is the coefficient of stiffness.

The total force in the *x* direction on the mass is

$$m\frac{d^2x}{dt^2} = -2T\sin\theta = 2k(L-a)\frac{x}{L},$$ where $\sin\theta = x / L$. Since $L^2 = d^2 + x^2$, the above equation can be written as

$$m\frac{d^2x}{dt^2}+2kx-\frac{2kax}{\sqrt{d^2+x^2}}=0$$

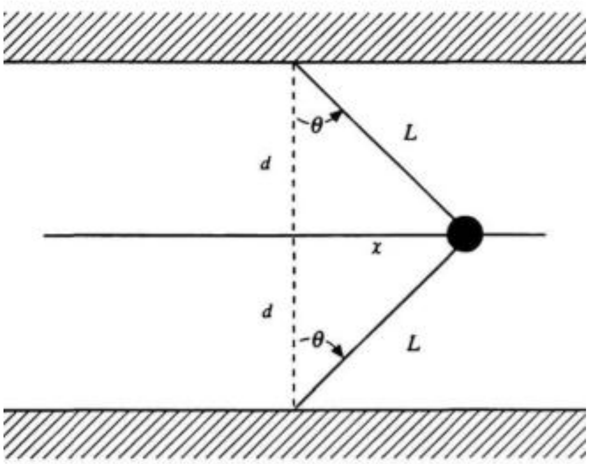

Fig-2

Vibrations of the Eardrum

The motion of the human eardrum is an important example of a nonlinear oscillation. The eardrum is constructed in an asymmetric form; its radial fibers experience much greater changes in tension when they undergo an oscillation of moderate amplitude toward the outside as compared to motion toward the inside.

For such motions, the eardrum can be modeled as an effective one-dimensional system for which the restoring force is approximated by

$$F(x)-ax-bx^2,$$

where *a* and *b* are positive constants. Consequently, the equation of motion is given by the following nonlinear differential equation

$$m\frac{d^2x}{dt^2}+ax+bx^2=0$$

where *m* is the "effective mass" of the eardrum. If the eardrum is acted on by several pure tones, then above becomes

$$m\frac{d^2x}{dt^2}+ax+bx^2=\sum_{i=1}^{n}A_i\cos(\omega_i\,\mathrm{t}+\theta_i)$$, where the (A_i,ω_i,θ_i) are constants.

Nonlinear Electrical Circuits

Electrical networks may be analyzed by applying Kirchhoff's laws [4]:

(1) The sum of the currents into (or away from) any point in the circuit is zero.

(2) Around any closed path of the circuit, the sum of the instantaneous voltage drops in a given direction is zero. The first law is just the statement that the current is the same throughout a simple electrical circuit.

To apply Kirchhoff's voltage law, it is necessary to know the contribution of each of the elements shown in Figure 3. These are given in the following rules:

i. The voltage drop across the resistance is *RI*.

ii. The voltage drop across the inductance is *Ldl/dt*. For an iron-core inductance coil, the voltage drop can be written in terms of the magnetic flux ϕ as $d\phi/dt$.

iii. The voltage drop across the capacitor is *Q/C*, where *Q* is the charge on the capacitor.

iv. The current in the circuit is *I = dQ/dt*.

Consider the problem of finding the variation in the flux ϕ for an iron-core inductance coil that is connected to a charged condenser as shown in Figure 4(a). If ϕ is the flux in the coil, then the equation of the circuit is

$$\frac{d\phi}{dt}+\frac{Q}{C}=0\ , \qquad (1)$$

where Q is the charge on the condenser and C is its capacitance. Applying rule-iv gives

$$\frac{d^2\phi}{dt^2}+\frac{I}{C}=0 \qquad (2)$$

In elementary circuit theory there exists a linear relationship between the current and flux: $I=\phi/L$, where L is the inductance. However, for an iron-core inductance, a more accurate expression of the relationship between current and flux (for values of flux that are not too large) is

$$I=A\phi-B\phi^3 \qquad (3)$$

where *A* and *B* are positive numbers. Substitution of this into Eq. (2) gives

$$\frac{d^2\phi}{dt^2}+\left(\frac{A}{C}\right)\phi-\left(\frac{B}{C}\right)\phi^3=0 \qquad (4)$$

This nonlinear differential equation has oscillatory solutions. The circuit of Figure 4(b) is described by the equation

$$\frac{d\phi}{dt}+RI+\left(\frac{1}{C}\right)Q=0 \qquad (5)$$

Using Eq. (3) and taking the derivative gives

$$\frac{d^2\phi}{dt^2}+R(A-3B\phi^2)\frac{d\phi}{dt}+\left(\frac{A}{C}\right)\phi-\left(\frac{B}{C}\right)\phi^3=0$$

which is another nonlinear differential equation having oscillatory solutions.

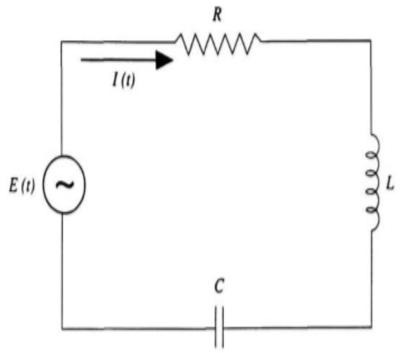

Fig-3

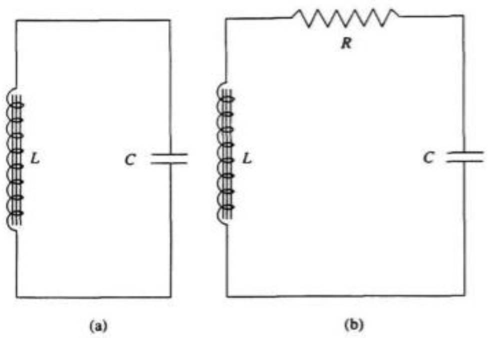

Fig-4

Now, a number of examples without derivation have been provided from various scientific fields that give rise to nonlinear equations.

Nonlinear Spring (Duffing oscillator): $\frac{d^2x}{dt^2}+x+\varepsilon x^3=0$

Negative-Resistance Oscillator (Van der Pol oscillator): $\frac{d^2x}{dt^2}+x=\varepsilon(1-x^2)\frac{dx}{dt}$

which arises in the study of triode vacuum tube oscillatory circuits.

Oscillations of a Diatomic Molecule: $m\frac{d^2y}{dt^2}=-ky+\left(\frac{k_1}{3}\right)y^2+\cdots$

Relativistic (an) harmonic oscillator: $\frac{d^2x}{dt^2}+\left[1-\left(\frac{dx}{dt}\right)^2\right]^{3/2}x=0$

IV. Application of Third Order Non-linear Differential Equation

Jerk oscillator: $\dddot{x} = \alpha\, x\, \dot{x}\, \ddot{x} - \beta\, \dot{x}\, \ddot{x}^2 - \gamma\, \dot{x} - \delta\, x^2\, \dot{x} - \varepsilon\, \dot{x}^3$

where an over-dot denotes the time derivative and the parameters $\alpha, \beta, \gamma, \delta$ and ε are given real constants.

Mulholland equation: $\dddot{x} + \ddot{x} + \dot{x} + x = \varepsilon\, f(x, \dot{x}, \ddot{x})$

where an over-dot denotes the time derivative and the parameter ε is given a real constants.

This equation arises in the study of control system.

These equations have also demonstrated their usefulness in ecology, economics and biology. That is a large number of problems in engineering and science can be formulated in the form of differential equation. Therefore, the solution of such problems lies essentially in solving the corresponding differential equations.

There are several analytical approaches to find approximate solutions to nonlinear problems, such as: Perturbation, Standard and modified Linstedt-Poincaré, Harmonic Balance, Homotopy, Iterative methods, etc.

Among them the most widely used method is the perturbation method where the nonlinear term is small. Harmonic Balance and iteration procedure are valid for small as well as large amplitude of oscillation, to obtain the periodic solution of such nonlinear problems.

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A note on the terms of undefined, does not exist, determinate, indeterminate forms and conflict of influences in the derivation of limit

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Abstract
In this short note the terms undefined, does not exist, meaningless, determinate, indeterminate and exerting conflict of influences of functions in the context of limit derivation are discussed by simple examples. Also, why the mathematical expression of the forms $\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, 0\times\infty, 1^{\infty}, 0^0, \infty^0$ are called indeterminate when evaluating limits are explained with the term "conflict of influences" of functions by providing some numerical examples and counter examples. Moreover, the indeterminate characteristics of functions are also shown graphically.

1. Introduction

We often encounter with the terms "undefined", "does not exist", "meaningless" and "indeterminate" in the various branches of mathematics. Particularly, in basic algebra division by zero is undefined and in the calculus the indeterminate term arises during the evaluation of limit of a function. The meanings of all of these terms differ from subject to subject in view of their contextual meaning.

In algebra, the statement "undefined or does not exist" means those is one that doesn't have one clear value. Some arithmetic operations may not assign a meaning to certain values of its opearands e.g. division by zero [1], [2]. In that case, the mathematical expression is called undefined respecting such operation. In topology, a topological space is defined as a set of points having certain properties, but in general geometry, the nature of these "points" is left entirely undefined. Similarly, in category theory a category consists of "objects" and "arrows"; again these are primitive, undefined terms. Another reason an expression might be undefined in the set theory, if a number is not in the domain of a function, it is said to be undefined for that number e.g. the function $f(x)=(1/x)$, which is undefined for $x=0$ and $f(x)=\sqrt{x}$ is undefined for negative values of x in the real number system [3]. Another word "meaningless" is used generally in elementary arithmetic, as a synonyms of the term "undefined or does not exist". A simple example is given to understand the word meaningless. Suppose one wish to distribute 10 apples among 5 persons. Each will get 2 apples and evenly can be distributed. Even if it requires distributing 10 apples among 4 people, the problem can be solved cutting the 2 apples into pieces, which introduces the idea of fractions $2\frac{1}{2}$. But, if the problem arises such as one has to distribute 10 apples among 0 people, cannot be solved in any way. There is no way to distribute 10 apples to nobody. So $10/0$, in elementary arithmetic, is said to be "meaningless". It is also be worthy replacing such nonexistence statement by the word "does not exist". As, division is the inverse of multiplication, the expression $a/0$ has no meaning in ordinary arithmetic. Since there is no number which, when multiplied by 0, gives a (assuming $a\neq 0$) and so division by zero is undefined. Again, since any number multiplied by zero is zero, the expression $0/0$ is also undefined. When, we discuss the limit of a function and if the undefined term $0/0$ arises, then it is called an "indeterminate form".[1],[2],[3] The term "indeterminate form" was first introduced by French physicist Moigno (1804-1884) in the middle of 19^{th} century who was a student of Cauchy. In the literature of evaluating limits, the other six indeterminate forms are $\frac{\infty}{\infty}, \infty-\infty, 0\times\infty, 1^{\infty}, 0^0, \infty^0$. It is important to note down that some undefined expressions are indeterminate i.e. the ultimate behavior of a function can't be determined and some undefined expressions are determinate whose limiting behavior can be evaluated by rewriting the limit problems algebraically or by using L'Hospital's rule which are found most of the calculus book [4]. Herein, we first introduce different fallacies if it is allowed in algebra the undefined term $a/0$ i.e. division by zero. Also, the fallacies of the indeterminate terms $0/0, 0^0$ will be discussed algebraically and graphically. Finally, the seven indeterminate terms will be explained by the words "conflict of influence" why to call them indeterminate.

3. Division by zero: (symbols $a/0,\ 0/0$ meaningless)

3.1 Early attempts

Probably, in earlier classes, most of the student confronted with the statement "division by zero is undefined or does not exist" [2]. Brahmagupta (598-668), an Indian mathematician, is the first who treated zero as a number defines operations involving zero in his text "*Brāhmasphutasiddhānta*" [7]. But in his texts, the author couldn't explain what would be the division by zero and his definition lead to algebraic absurdities. According to Brahmagupta "A positive or negative number when divided by zero is a fraction with the zero as denominator. Zero divided by a negative or positive number is either zero or is expressed as a fraction with zero as numerator and the finite quantity as denominator. Zero divided by zero is zero". In the 9th century another Indian mathematician, Mahāvīra unsuccessfully tried to correct Brahmagupta's mistake in his book in *Ganita Sara Samgraha* "A number remains unchanged when divided by zero".

3.2 Fallacies of the expression like $a/0,\ 0/0$

When working with numerical computation, division by zero is not allowed. If it were allowed many bizarre and absurd results would arise. As in our earlier section, it is noted that undefined expression is one which does have a single clear rather assumes many values. Herein, we discuss some fallacies if it is allowed to division be zero [1-3].

i). From the properties of number system it is known that division is the inverse of multiplication. If $b \neq 0$ and $\frac{a}{b} = c$ then it is equivalent to $a = b \times c$. If we assume that $b = 0,\ \frac{a}{0} = c$ then it must be $a = 0 \times c$. From this, it is clear that there is no single value of c which satisfies the algebraic expression $a = 0 \times c$, rather c assumes any numbers. Therefore, we can't assign any real value to $\frac{a}{0}$. Even, if we assume both $a = b = 0$ then $\frac{0}{0} = c$ implies that $0 = 0 \times c$. Every number solves this equation instead of there being a single number. In general, a single value can't be assigned to a fraction where the denominator is zero.

ii) In basic algebra, $\frac{a}{a} = 1$, so if we presume $\frac{0}{0} = 1$ then $0 \times 1 = 0$ and $0 \times 2 = 0$ lead to $0 \times 1 = 0 \times 2 = 0$. Dividing by zero gives

$$\frac{0 \times 1}{0} = \frac{0 \times 2}{0}$$
$$\frac{0}{0} \times 1 = \frac{0}{0} \times 2$$
$$1 \times 1 = 1 \times 2$$
$$1 = 2$$

The disguised operation divides by zero consequences this absurd result.

3.2 Some fallacies of the indeterminate form 0^0

Some mathematicians consider that 0^0 is equal to 1 as zero power of any number is equal to 1. In the following examples we see that this is not true rather some fallacies arise [5],[8-10].

i) Let's check the calculation: $x^0 = x^{1-1} = x^1 . x^{-1} = \frac{x}{x} = 1$. Now if we just plug in $x = 0$, we get $0^0 = 1$.

ii). $0^x = 0^{1+x-1} = 0^1 \times 0^{x-1} = 0$, which is true since anything times 0 is 0. Again, replacing $x = 0$, we get $0^0 = 0$ which results different from the previous one.

iii). Now, let's check value of the limit function:

$$\begin{aligned}
\lim_{x\to 0^+} x^x &= \lim_{x\to 0^+} \exp(\log x^x) \\
&= \exp\left(\lim_{x\to 0^+} x\log(x)\right) \\
&= \exp\left(\lim_{x\to 0^+} \frac{\log(x)}{x^{-1}}\right) \\
&= \exp\left(\lim_{x\to 0^+} \frac{\frac{d}{dx}(\log(x))}{\frac{d}{dx}(x^{-1})}\right) \\
&= \exp\left(\lim_{x\to 0^+} -x\right) \\
&= \exp(0) \\
&= 1
\end{aligned}$$

So, since $\lim_{x\to 0^+} x^x = 1$, again substituting $x = 0$ it results $0^0 = 1$.

iv). Have a look the properties the function: $f(x, y) = y^x$ for positive integers x and y. Here

$$y^x = 1\times y\times y\times y\times\cdots\times y$$

where the y is repeated x times. If the value of x is one, then y is repeated merely one time, so we get

$$y^x = 1\times y$$

However, if x is zero, y is repeated just zero times, yielding

$$y^0 = 1$$

which holds for any y. Hence, when $y = 0$, it results

$$0^0 = 1$$

v). Consider the binomial theorem $(a+b)^x = \sum_{k=0}^{\infty}\binom{x}{k} a^k b^{x-k}$, where $\binom{x}{k}$ denotes the binomial coefficients. Now, letting $a = 0$ and $b \neq 0$, we get

$$\begin{aligned}
b^x = (0+b)^x &= \sum_{k=0}^{\infty}\binom{x}{k} 0^k\, b^{x-k} \\
&= \binom{x}{0}0^0\, b^x + \binom{x}{1}0^1\, b^{x-1} + \binom{x}{2}0^2\, b^{x-2} + \binom{x}{3}0^3\, b^{x-3} + \cdots \\
&= \binom{x}{0}0^0\, b^x, \text{ assume that } 0^k = 0 \text{ for } k > 0 \\
&= 0^0\, b^x
\end{aligned}$$

Therefore, $b^x = 0^0\, b^x$ implies that $0^0 = 1$. Hence, if we don't consider $0^0 = 1$ then the binomial theorem does not hold.

4. Graphical representation of indeterminate form $0/0$

In calculus, particularly the indeterminate forms $0/0, 0^0$ are very common because it often arises in the evaluation of derivatives using their limit definition. Typically, the limiting behavior of a function is determined in three ways: numerical approach by constructing a table of values; graphical approach by drawing the function and analytical approach by using algebra of calculus. Let we explore limiting behavior of the following functions $\lim_{x\to 0}\frac{x}{x}$, $\lim_{x\to 0}\frac{x^2}{x}$, $\lim_{x\to 0}\frac{x}{x^3}$ graphically all of whose are indeterminate forms of type $0/0$.

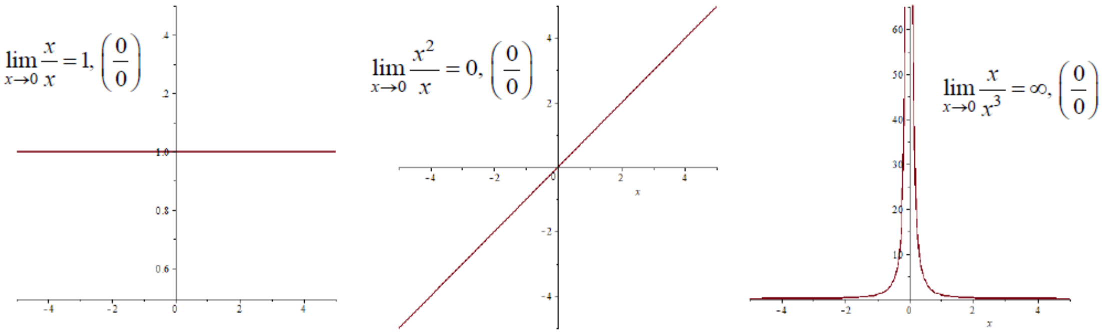

Figure 1. (a) The graph of $y=\frac{x}{x}$ (b) The graph of $y=\frac{x^2}{x}$ (c) The graph of $y=\frac{x}{x^3}$

From the Figs. 1(a-c), it is apparent that as x approaches 0, the ratios $\frac{x}{x},\frac{x^2}{x},\frac{x}{x^3}$ go to 1, 0 and ∞ respectively. In each case, on substituting the limits, the expression results in the form of $0/0$, which is undefined. In general speaking, $0/0$ may take on the values 1or 0 or ∞. Therefore, the limit

$$\lim_{x\to c}\frac{f(x)}{g(x)}$$

when the functions $f(x)$ and $g(x)$ both approach 0 as x approaches some limit point does not have enough information to evaluate the limit.

5. Graphical representation of indeterminate form 0^0:

Let’s we check the graphical behavior of the following functions $\lim_{x\to 0^+}0^x$, $\lim_{x\to 0^+}x^0$ as x approaches 0 whose pattern of type 0^0.

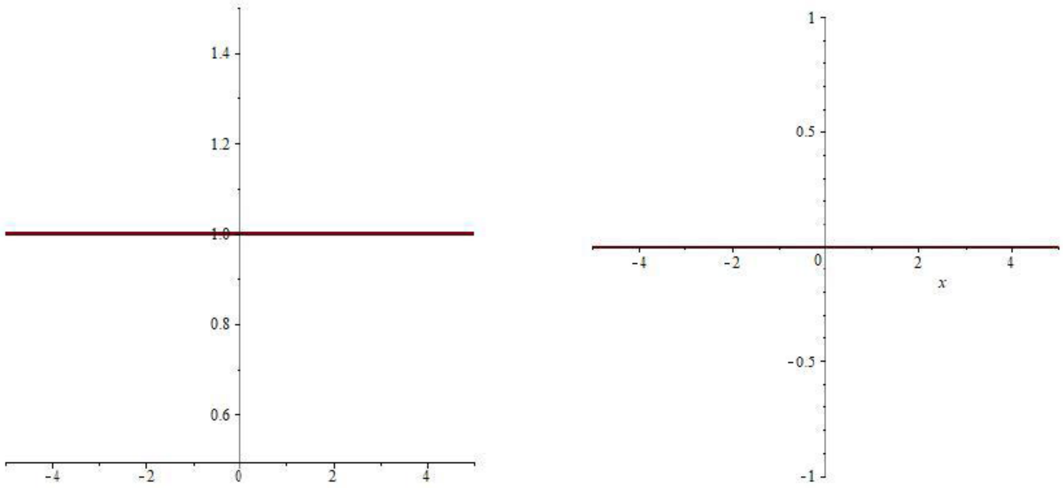

Figure 2. (a) The graph of $y=x^0(0^0)$ (b) The graph of $y=0^x(0^0)$

From the above Figs. 2(a-b), it is seen that

$$\lim_{x\to 0^+}0^x=1 \text{ and } \lim_{x\to 0^+}x^0=0$$

i.e. the function x^0 tends to 1 but the function 0^x tends to 0 as $x\to 0^+$, both of which in the indeterminate form 0^0. So, the indeterminate form 0^0 may possess different values. Thus, in evaluating the limit of the form

$$\lim_{x\to c}f(x)^{g(x)}$$

as $\lim_{x\to c}f(x)=0$ and $\lim_{x\to c}g(x)=0$ does not provide enough information to calculate the limit.

6. Evaluating indeterminate forms algebraically

In the table below, the seven indeterminate forms and similar type some determinate forms are presented.

Table 1: List of indeterminate forms ($a \neq 0, c \neq 1, c > 0$)

	Indeterminate forms	Determinate forms
1	$\frac{0}{0}$	$\frac{a}{0} \to \pm\infty$ (The sign of ∞ depends on both the sign of a and the sign of denominator)
2	$\frac{\pm\infty}{\pm\infty}$	$\frac{b}{\pm\infty} \to 0,\ \frac{\pm\infty}{b} \to \infty$
3	$\infty - \infty$	$\infty + \infty \to \infty, \quad -\infty - \infty \to -\infty$
4	$0 \cdot \pm\infty$	$a \cdot \pm\infty \to \pm\infty$ (The sign of ∞ depends on both the sign of a and the sign of the second factor) $\pm\infty \cdot \pm\infty \to \pm\infty$ (The sign of ∞ depends on the signs of both factors)
5	0^0	$0^{\infty} \to 0$ $0^{-\infty} \to \pm\infty$ (The sign of ∞ depends on whether the form approaches zero on the left or on the right)
6	$1^{\pm\infty}$	$c^{\infty} \to \infty$, when $c > 1$ $c^{-\infty} \to 0$, when $c > 1$ $c^{\infty} \to 0$, when $0 < c < 1$ $c^{-\infty} \to \infty$, when $0 < c < 1$
7	∞^0	$\infty^a \to \infty$ $\infty^{\infty} \to \infty$ $\infty^{-\infty} \to 0$

In this section, we will see that the adjective “indeterminate” does not imply that the limit does not exist by some examples. Two theorems are presented which are needed to evaluate the indeterminate forms.

Theorem 1: Let

$$f(x) = \frac{p(x)}{q(x)}$$

be a rational function, and let a be any real number.

(a) if $q(a) \neq 0$, then $\lim_{x \to a} f(x) = f(a)$.

(b) if $q(a) = 0$, but if $p(a) \neq 0$, then $\lim_{x \to a} f(x)$ does not exist

Theorem 2: (L'Hospital's rule for form 0/0) Suppose that f and g are differentiable functions on an open interval containing $x = a$, except possibly at $x = a$ and that $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}.$$

Before going to evaluate the indeterminate problems, let's take a moment to clarify some notation.

6.1 Notation: The role of 0 and ∞ during the evaluation of limits

It is important to remember that when we are evaluating a limit by substituting in c for x we are not really plugging in the value c exactly, but rather we are substituting the values that are arbitrarily close to c but not equal to c. If $f(x) \to 0/0$, it means that both the numerator and denominator of $f(x)$ are shrinking in magnitude as x gets closer and closer to c. Thus, the “0” in the expression are not really zeros on the real line rather they stand for numbers that are very very close to zero. Again if $f(x) \to \infty - \infty$, we don’t actually mean the subtraction of ∞. Remember that ∞ is not a real number it just explain the unbounded behavior of the function where the magnitude of the numbers increasing without bound. When evaluating limit, both 0 and ∞ express the behavior of the function around $x = c$ whereas 0 stands for some number that is arbitrarily close to zero; $+\infty$ stands for some number that is arbitrarily large.

7. Calculating limits algebraically:

7.1. Indeterminate forms (0/0)

From the Tab. 1, it is noticed that $0/0$ is indeterminate form whereas $a/0,\ (a \neq 0)$ is determinate form. In this section we will explain it details why $a/0,\ (a \neq 0)$ is determinate but $0/0$ is not, providing some examples [6].

Table 2: examples of indeterminate forms (0/0)

1. $\lim\limits_{x \to -2^+} \frac{x}{x+2}$	$f(c) = \frac{a}{0},\ a \neq 0$
2. $\lim\limits_{x \to 0} \frac{\sqrt{x+1}-1}{x}$	$f(c) = \frac{0}{0}$, but the actual limit of $f(x) \to$ non-zero number as $x \to c$
3. $\lim\limits_{x \to 0} \frac{x^2}{\sqrt{x+1}-1}$	$f(c) = \frac{0}{0}$, but the actual limit of $f(x) \to 0$ as $x \to c$
4. $\lim\limits_{x \to 0} \frac{\sqrt{x+1}-1}{x^2}$	$f(c) = \frac{0}{0}$, but the actual limit of $f(x) \to \pm\infty$ as $x \to c$

Example 1: $\lim\limits_{x \to -2^+} \frac{x}{x+2} = \frac{-2^+}{0^+} = -\infty$.

In this case, as x approaches -2 from the right, the numerator of $f(x)$ becomes very close to -2 while the denominator's magnitude grows ever smaller. Consequently, $f(x) \to -\infty$ as $x \to -2$.

Example 2: $\lim\limits_{x \to 0} \frac{\sqrt{x+1}-1}{x}$, $\left(\frac{g(0)}{h(0)} = \frac{0}{0}\right)$, rationalizing the terms, we get

$$\begin{aligned}\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} &= \frac{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}{x.(\sqrt{x+1}+1)} \\ &= \frac{1}{(\sqrt{x+1}+1)} \\ &= \frac{1}{(\sqrt{0+1}+1)} = \frac{1}{2}.\end{aligned}$$

In this case, $f(x)$ approaches (0/0) as x approaches 0 i.e. the magnitudes of both numerator and denominator of $f(x)$ grow ever smaller. But after rationalizing we may conclude the value of the limit. The former indeterminate (0/0) behavior does not provide enough information to conclude anything about the limit. Here we examine this problem providing numerical evidence as well as coining a new term "conflict of influence" in the literature of evaluating limit:

Let $g(x) = \sqrt{x+1}-1$ and $h(x) = x$. As $x \to 0$ the numerator $g(x) \to 0$ from the left and right side of 0 in the following way:

$$-0.9,\ -0.8,\ -0.7,\ -0.6,\ -0.5,\ -0.4,\ -0.3,\ -0.2,\ -0.1 \cdots \to 0 \tag{1}$$

$$0.9,\ 0.8,\ 0.7,\ 0.6,\ 0.5,\ 0.4,\ 0.3,\ 0.2,\ 0.1 \cdots \to 0 \tag{2}$$

Also $h(x) \to 0$ as $x \to 0$ say $h(x)$ approaches zero from the left and right side of 0 as:

$$-0.91, -0.81, -0.71, -0.61, -0.51, -0.41, -0.31, -0.21, -0.11 \cdots \to 0 \tag{3}$$

$$0.91,\ 0.81,\ 0.71,\ 0.61,\ 0.51,\ 0.41,\ 0.31,\ 0.21,\ 0.11 \cdots \to 0 \tag{4}$$

Suppose $g(x)$ and $h(x)$ are following the pattern (1) and (3), then the ratio $f(x) = \frac{g(x)}{h(x)}$ tends to $+\infty$ or 0 and from the pattern (2) and (3) the ratio tends to $-\infty$ or 0. Similarly, if we consider (1), (4) or (2), (4) the ratio tends to $-\infty$ or ∞ or 0 or vice versa. Thus, it is important to know about what the relationship exist between the numerator and the denominator. It depends on how "small" or "large" the magnitude of the numerator in comparison to the denominator. This is called the "conflict of influence" of functions.

Example 3: $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt{x+1}-1}\left(\dfrac{0}{0}\right)$

Rationalizing we get, $\lim\limits_{x\to 0} x(\sqrt{x+1}+1) \to 0(1+1)=0$

Example 4: $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}-1}{x^2}\left(\dfrac{0}{0}\right)$

Rationalizing we get, $\lim\limits_{x\to 0}\dfrac{1}{x(\sqrt{x+1}+1)} \to \dfrac{1}{0(\sqrt{1}+1)}=\dfrac{1}{0}$.

If $x\to 0^-$ then this limit tends to $-\infty$ and if $x\to 0^+$ then this limit tends to ∞.

Differences between examples 1,2,3 and 4

Let's we summarize these four examples and draw differences among them. In each of case, the problem is a rational function where the denominator is zero when substituted the limit in the rational function. In each case, the numerator and denominator had different relationships as $x\to c$.

Remark 1: when $f(c)=a/0$ for some $a\neq 0$, (Example-1) then this provides enough information to tell us that $f(x)\to\pm\infty$ as $x\to c$.

Remark 2: when $f(c)=0/0$ (Example-2,3,4) then this does give us enough information to tell us about what happens to $f(x)$ as $x\to c$ because it does not tell us anything about the relationship between the numerator and denominator. In these cases, both the numerator and denominator are shrinking in magnitudes but we don't know their shrinking rate whether shrinking at the same rate or shrinking "much faster" than the other or vice versa. Therefore, the ratio of the numerator to the denominator produces conflict of influences of the functions of their shrinking resulted towards zero or increasing or decreasing without bound [6].

Remark 3: Also, noticed that these definitions only have meaning when we are calculating limit. In elementary algebra, the answer of both the expression $a/0, 0/0$ would be undefined and it would be incorrect to say anything about determinate or indeterminate because not calculating limit [6].

Here, we are noticed that $a/0$ (when $a\neq 0$) and $0/0$ both are undefined but the former $a/0$ tells us something about the limit behavior even though it is undefined, while the latter $0/0$ doesn't provide us useful information to conclude about the limit which lead to the definitions of determinate and indeterminate forms.

Definition: Determinate and indeterminate forms:

Suppose we have the limit problem $\lim\limits_{x\to c} f(x)$, where c is substituted for x produces an undefined expression for $f(c)$: That undefined expression is called determinate if it gives enough information to determine the limit behavior around $x=c$ and that undefined expression is called indeterminate form if there is more than one possible type of limit behavior around $x=c$ and so will have to do further calculations to figure out.

7.2 Indeterminate forms of type (∞/∞)

Table 3: examples of indeterminate forms (∞/∞)

5. $\lim\limits_{x\to -\infty}\dfrac{3}{2x-5}$	$f(c)=\dfrac{a}{\pm\infty}$
6. $\lim\limits_{x\to \infty}\dfrac{2x^2}{e^{-x}}$	$f(c)=\dfrac{\pm\infty}{b}$, yields the undefined expression as $x\to c$
7. $\lim\limits_{x\to -\infty}\left(\dfrac{4x^2-1}{2x^3+1}\right)$	$f(c)=\dfrac{\pm\infty}{\pm\infty}$, undefined but the actual limit of $f(x)\to 0$ as $x\to c$
8. $\lim\limits_{x\to -\infty}\left(\dfrac{4x^2-1}{2x^2+1}\right)$	$f(c)=\dfrac{\pm\infty}{\pm\infty}$, undefined but the actual limit of $f(x)\to\pm\infty$ as $x\to c$

Example 5: $\lim\limits_{x\to-\infty}\dfrac{3}{2x-5}=\lim\limits_{x\to-\infty}\dfrac{\frac{3}{x}}{2-\frac{5}{x}}\to\dfrac{0}{2-0}=0$

Example 6: $\lim\limits_{x\to\infty}\dfrac{2x^2}{e^{-x}}\to\dfrac{2(\infty)^2}{e^{-\infty}}\to\infty$

Example 7: $\lim\limits_{x\to-\infty}\left(\dfrac{4x^2-1}{2x^3+1}\right)\left(\dfrac{-\infty}{\infty}\right)=\lim\limits_{x\to-\infty}\dfrac{\frac{4}{x}-\frac{1}{x^3}}{2+\frac{1}{x^3}}\to\dfrac{0-0}{2+0}=0$

Example 8: $\lim\limits_{x\to-\infty}\left(\dfrac{4x^3-1}{2x^2+1}\right)\left(\dfrac{-\infty}{\infty}\right)=\lim\limits_{x\to-\infty}\dfrac{4-\frac{1}{x^3}}{\frac{2}{x}+\frac{1}{x^3}}\to\dfrac{4-0^-}{0^-+0^-}=-\infty$

Following observations can be summarized from the last four examples:

Remark 4: when $f(c)=b/\pm\infty$ then this provide good enough information to tell us that $f(x)\to 0$ as $x\to c$. Here, the numerator is fixed but the denominator increasing in magnitude and results in numbers ever closer to zero.

Remark 5: when $f(c)=\pm\infty/b$ also this expression provides enough information to tell us that $f(x)\to\pm\infty$ as $x\to c$. Here, dividing a value with ever increasing results in numbers with larger and larger magnitude which tends to unbounded.

Remark 6: when $f(c)=\pm\infty/\pm\infty$ this doesn't tell us enough information about what happens to $f(x)$ as $x\to c$ because it does not tell us anything about the relationship between the numerator or denominator. Both the numerator and denominator are growing without bound, but we don't know if they are growing at the same rate (the ratio will be fixed), or the magnitude of one of them is growing "much faster" than the other (either shrinking towards zero or increasing or decreasing without bound) as a result produces a "conflict of influences" of the functions.

7.3 Indeterminate forms of type $(\infty-\infty)$

From the third row of Tab. 1, $\infty-\infty$ is indeterminate, whereas $\infty+\infty$ or $-\infty-\infty$ are determinate. Why this is the case we will see by the following examples:

Table 4: examples of indeterminate forms $(\infty-\infty)$

9. $\lim\limits_{x\to\infty}(x+\sqrt{x})$	$f(c)=\infty+\infty$, so $f(x)\to\infty$ as $x\to c$
10. $\lim\limits_{x\to\infty}(x-\sqrt{x})$	$f(c)=\infty-\infty$, so $f(x)\to\infty$ as $x\to c$
11. $\lim\limits_{x\to\infty}(\sqrt{x}-x)$	$f(c)=\infty-\infty$, so $f(x)\to-\infty$ as $x\to c$
12. $\lim\limits_{x\to\infty}(x-\sqrt{x^2+5})$	$f(c)=\infty-\infty$, so $f(x)\to 0$ as $x\to c$

Example 9: $\lim\limits_{x\to\infty}(x+\sqrt{x})\to\infty+\sqrt{\infty}=\infty+\infty=\infty$.

Example 10: $\lim\limits_{x\to\infty}(x-\sqrt{x}),\ \ (\infty-\infty)$

$$=\lim_{x\to\infty}\frac{(x-\sqrt{x})}{1}.\frac{(x+\sqrt{x})}{(x+\sqrt{x})}=\lim_{x\to\infty}\frac{(x^2-x)}{(x+\sqrt{x})}=\lim_{x\to\infty}\frac{x-1}{1+\sqrt{\frac{1}{x}}}\to\frac{\infty-1}{1+\sqrt{\frac{1}{\infty}}}=\infty.$$

Example 11: $\lim\limits_{x\to\infty}(\sqrt{x}-x),\ \ (\infty-\infty)\to-\infty$

Example 12: $\lim_{x\to\infty}(x-\sqrt{x^2+5}),\ \ (\infty-\infty)\to 0$

From the above examples we may lead to the following conclusions:

Remark 7: From example 9, it is seen that $\infty+\infty$ approaches toward ∞ because adding two values together, both of are increasing without bound and resulted output increase without bound. Similarly, the combined effect of the expression $-\infty-\infty$ is decreasing without bound.

Remark 8: From the example 10 the limiting value is ∞ as the magnitude of the first term grows "more quickly" than the magnitude of the second term, on the contrary, in example 11, the limiting value is $-\infty$ as the magnitude of the second term grows "faster" than the first term. But, it is interesting that in example 12, the limiting value is 0, since the magnitudes of both the first and second terms grow at about "the same" rate. Therefore, when the functions limiting form of type $(\infty-\infty)$, then the resultant may be ∞ or $-\infty$ or 0. Here, we provide numerical examples of type $(\infty-\infty)$, why do they exert conflicting influence?

Suppose the first and second functions are increasing in the following way

$$1,\ 4,\ 8,\ 16,\ 32,\ 48,\ \ 64,\ \cdots$$
$$1,\ 5,\ 9,\ 19,\ 35,\ 50,\ 66,\ \cdots$$

Then the difference of the corresponding terms of the two sequences $0,-1,-1,-3,-3,-6,-6,\cdots\to-\infty$ But, if the increasing order follows reversely the difference the terms will be the sequence $0,\ 1,\ 1,\ 3,\ 3,\ 6,\ 6,\ \cdots\to+\infty$, even if both the functions increase with the same rate the resultant sequence will be $0,\ 0,\ 0,\ 0,\ \cdots\to 0$

This numerical evidences show that the functions exert conflicting influences so, until we know more about the relationship between the first and the second value in the expression $\infty-\infty$, we can't conclude the ultimate limiting behavior of the function.

7.4 Indeterminate form of type $0\cdot\pm\infty$

In the fourth row of Table 1, it is noticed that $0\cdot\pm\infty$ is indeterminate and $0\cdot\pm\infty$ and $\pm\infty\cdot\pm\infty$ are determinate. In this section, we will discuss difference between the expression $0\cdot\pm\infty$ and the two cases $a\cdot\pm\infty$ and $\pm\infty\cdot\pm\infty$. Let's we see why $0\cdot\pm\infty$ is indeterminate by some concrete numerical examples. Here, the first factor zero is not algebraic absolute value zero, it stands to express the behavior of a function which tends to zero and the second factor $\pm\infty$ stands to express the behavior of a function which increases or decreases without bound.

- If the magnitude of the first factor is decreasing "much faster" than the magnitude of the second factor increasing e.g.

$$1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{1}{16},\ \cdots\to 0\ \ \text{and}\ \ -1,\ -4,\ -16,\ -64,\ -256,\ \cdots\to-\infty$$

then multiplying each term from the first sequence with each term from the second sequence gives us:

$$1.(-1),\ \frac{1}{2}(-4),\ \frac{1}{4}(-16),\ \frac{1}{8}(-64),\ \frac{1}{16}(-256),\ \cdots=-1,\ -2,\ -4,\ -8,\ -16,\ \cdots\to-\infty$$

In this case, the expression $0\cdot\pm\infty$ tends toward $\pm\infty$.

- If the magnitude of the first factor is decreasing "much faster" than the magnitude of the second factor increasing likewise below:

$$1,\ -\frac{1}{4},\ -\frac{1}{16},\ -\frac{1}{64},\ -\frac{1}{256},\ \cdots\to 0\ \ \text{and}\ \ 1,\ 2,\ 4,\ 8,\ 16,\ \cdots\to\infty$$

then the product of terms of the first factor with the terms of second factor yields

$$-1.1,\ -\frac{1}{4}.2,\ -\frac{1}{16}.4,\ -\frac{1}{64}.8,\ -\frac{1}{256}.16,\ \cdots\ =\ -1,\ -\frac{1}{2},\ -\frac{1}{4},\ -\frac{1}{8},\ -\frac{1}{16},\ \cdots\to 0.$$

In this case, the term $0\cdot\pm\infty$ tends toward 0 .

- Another situation may also arise if the first factor is decreasing at "about the same" rate that the magnitude of the second factor is increasing such as

$$1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{1}{16},\ \cdots \to 0 \ \text{ and } \ 1,\ 2,\ 4,\ 8,\ 16,\ \cdots \to \infty$$

then the product of corresponding terms of from the two sequences results

$$1.1,\ \frac{1}{2}.2,\ \frac{1}{4}.4,\ \frac{1}{8}.8,\ \frac{1}{16}.16,\ \cdots\ = 1,\ 1,\ 1,\ 1,\ 1,\ \cdots \to 1$$

In which case, $0\cdot\pm\infty$ converges to some other fixed value e.g. 1.

Remark 9: From the above three examples, it is evident that the relationships between the first and second value in the expression $0\cdot\pm\infty$ is important to know to be confirm or conclude the behavior of the function of $f(x)$ as x tends to a value say c.

7.5 Indeterminate form of type 0^0

The expression 0^0 is actually not undefined, which we have shown in our earlier section that $0^0 = 1$ or $0^0 = 0$. However, this is not really relevant to the study of limit in calculus. When we are finding the limit of $f(x)$, we are not getting 0^0 is exactly, instead we wish to determine the behavior of $f(x)$ as it tends towards 0^0. In the Tab. 1, 0^0 is indeterminate whereas $0^{\pm\infty}$ is determinate. Why $0^{\pm\infty}$ is determinate it will be discussed firstly in this section and secondly the indeterminate form 0^0 will be outlined.

We see that $0^{\pm\infty}$ tends to 0 because multiplying some value whose magnitude is very very close to 0 (suppose 0.000001) by itself a greater and greater number of times will just yield a third value whose magnitude is also decreasing indefinitely i.e. $(0.000001)^{\text{very largest no.}} \to 0$.

However, if we think about 0^0, we dive into the problem that we don't know the relationship between the base and the exponent.

Remark 10: It may happen that the magnitude of the base decreases "much faster" than the magnitude of the exponent. In this case 0^0 would tend toward 0. Also if it could be that the magnitude of the exponent decreases "much faster" than the magnitude of the base then 0^0 would tend toward 1 and exhibits conflict of influences. Sharing with our previous examples, until we know the relationship between the base and exponent we can't conclude the limit behavior of $f(x)$ as $x \to c$.

7.6 Indeterminate form of type $1^{\pm\infty}$

The sixth indeterminate form in Tab. 1 is $1^{\pm\infty}$. In this section, succinctly we outline why $1^{\pm\infty}$ is indeterminate and $c^{\pm\infty}$ is determinate.

It is clear that when $c > 1$, c^{∞} tends toward ∞. Again, if $0 < c < 1$, c^{∞} tends toward to 0. Reversely, when $c > 1$, $c^{-\infty}$ tends toward 0 and when $0 < c < 1$, $c^{-\infty}$ tends toward to ∞. However, if we think about $1^{\pm\infty}$, we run into the problem that we don't know the relationship between the base and exponent (like as indeterminate form 0^0)

Remark 11: if the base tends "much more quickly" towards 1 than the magnitude of the exponent tends toward infinity in that case $1^{\pm\infty} \to 1$. If the exponent tends toward positive infinity "more quickly" than the base tends toward 1 and the base approaches 1 from the positive side (base greater than 1) $1^{\pm\infty} \to \infty$ and if the base approaches 1 from the negative side (base less than 1) $1^{\pm\infty} \to 0$. It could be that the exponent tends toward negative infinity "more faster" than the base tends toward 1 and the base approaches 1form the right side, in this case $1^{-\infty} \to 0$ and from the negative side $1^{-\infty} \to \infty$. Therefore, $1^{\pm\infty}$ does not have one clear value and shows conflicting of influence.

Conclusion: In this short note, we discuss the meaning of the terms undefined, does not exist, meaningless, determinate, indeterminate forms, conflict of influences in the literature of mathematics. When we evaluate the limiting behavior of a function the word "undefined" turns into "indeterminate" form. But, not every undefined algebraic expression corresponds to an indeterminate form. Some algebraic manipulations or by applying L'Hospital's rule or by other techniques many indeterminate forms can be converted into determinate form. Why the mathematical expressions $\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, 0\times\infty, 1^{\infty}, 0^{0}, \infty^{0}$ are considered indeterminate is explained by introducing the terms "conflict of influence" behavior of functions with numerical examples.

Acknowledgement: I would like to acknowledge the author of the article [6].

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Applications of Soliton Solutions in Different Fields of Mathematical Sciences

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The study of the traveling wave solutions of nonlinear partial differential equations (both classical and fractional) play a vital role in describing the characters of nonlinear problems in the area of mathematical physics, physical sciences, engineering and applied science. Many nonlinear wave systems have been investigated usually to express numerous problems in mathematical physics and physical sciences, such as, the phenomena of flow of heat, propagation of wave and shallow water waves, plasma physics, optical fibers, chemical kinematics, electricity, fluid mechanics, biology and quantum mechanics, etc. Therefore, constructing exact solitary wave solutions for different types of nonlinear wave problems are a modern research area in nonlinear sciences. Various complex nonlinear phenomena in different fields of nonlinear sciences can be expressed in the form of nonlinear partial differential equations (NPDEs). Soliton solutions of are very much significant to the researches and recently many researchers paid attention to investigate the exact traveling wave solutions of NPDEs in different fields.

In mathematical physics, a soliton or solitary wave or traveling wave solutions is a self-reinforcing wave packet that keeps its shape when it disseminates at a constant velocity. Solitons are basis by a cancellation of nonlinear and dispersive effects in the medium and are the solutions of a prevalent class of weakly nonlinear dispersive partial differential equations recounting physical phenomena.

The soliton phenomenon was first described in 1834 by John Scott Russell [1] (1808–1882) who observed a solitary wave in the Union Canal in Scotland. He reproduced the phenomenon in a wave tank and named it the "Wave of Translation". He has rapidly found applications in various fields of Mathematical Physics, Bio-Physics, Engineering and Data Communications. Hardly any ground has been left intact in the short length of forty years from its discovery.

This special matter is therefore to scratch the growing importance of this field and to put together the works of principal researchers in the area in one place for later orientation. Solitons and applications base on this idea may transform our way of living. He used up some time making practical and theoretical investigations of these waves and built wave tanks at his home and observed some key properties [1, 2]:

- The waves are stable, and can travel over very large distances.
- The speed depends on the size of the wave, and its width on the depth of water.
- Unlike normal waves they will never merge, so a small wave is overtaken by a large one, rather than the two combining.
- If a wave is too big for the depth of water, it splits into two, one big and one small.

After then, in 1895 Diederik Korteweg and Gustav de Vries [3] provided the solitary wave and periodic cnoidal wave solutions. Furthermore, in literature, different solitons are investigated by several scientists time by time.

In 1973, Akira Hasegawa of AT&T Bell Labs was the first to suggest that solitons could exist in optical fibers and in 1997, M. Gedalin, T.C. Scott, and Y.B. Band, introduced the Optical Solitons [4]. They also proposed the idea of a soliton-based transmission system to increase performance of optical telecommunications.

In 1987, Emplit, et al. [5] from made the first experimental observation of the propagation of a dark soliton, in an optical fiber.

In 1998, A.M. Kosevich, *V.V. Gann, A.I. Zhukov and V.P. Voronov [6], suggested Magnetic soliton motion in a nonuniform magnetic field.*

In 2006, W. Craig, P. Guyenne, J. Hammack, D. Henderson, and C. Sulem [7], introduced water wave soliton in fluid mechanics.

In 2008, D.Y. Tang et al. *[8],* observed a novel form of higher-order vector soliton from the perspectives of experiments and numerical simulations.

Different types of Solitons are arises in different fields including mathematical physics, such as, wave phenomenon of a few rivers including the river Severn are named by a train of solitons. The solitons occur as the undersea internal waves, Atmospheric solitons , the soliton model in neuroscience proposes to explain the signal conduction within neurons as pressure solitons, soliton stability is due to topological constraints, much experimentation has been done using solitons in fiber optics, soliton for water wave applications and solitons in a fiber optic system are described by the Manakov equations.

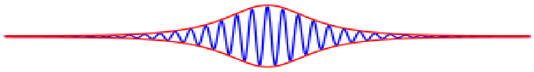

Fig.: Soliton

The solitons are available used in different fields in Mathematical sciences and the soliton solutions are typically obtained by means of the inverse scattering transform, and owe their stability to the inerrability of the field equations. Modified Liouville and the regularized long wave equations *[15, 16]*, are used in physics to handle the nonlinear waves system and making use to transform into ordinary nonlinear form. The computer symbolic systems, likely, Maple and Mathematica allow us to perform complicated and tedious calculation.

Different well known shape are established trough the traveling wave solution (solitons) of NLDES, inasmuch as, kink shape soliton, belled shape soliton, singular kink shape soliton, periodic wave shape soliton, singular periodic shape wave soliton, etc. which are shown below [9]:

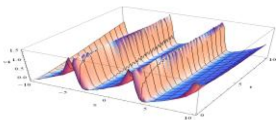

Fig.: Bell shape Soliton

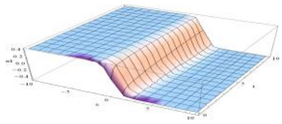

Fig.: Kink shape Soliton

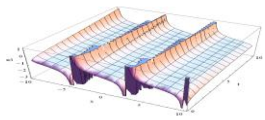

Fig.: Singular Periodic Shape Soliton

Several influential methods have been developed for the solitary waves solution of nonlinear partial equation such as the homogeneous balance method, as for instance, the new generalized the (G'/G)-expansion method [9], the first integral method [10], the fractional sub-equation method [11], the Exp-expansion method [12], the variational iteration method [13], the MSE method [14], etc.

The obtained soliton solutions by applying the above methods for nonlinear partial evolution equations [9-14] (for minimalism the all references are not shown here) have several applications as remarkable to models in physical science, mathematical physics, engineering, such as, the water waves gravity in the long-wave regime, the fractional quantum mechanics, the shallow water waves under gravity and propagation waves, the fluid flow of liquid through foam arisen by gravity and capillarity, the electro-hydro-dynamical model for local electric field, the ion acoustic waves in plasma and signal processing waves through optical fibers, the solitary waves in a density and current stratified shear flow with a free surface and the waves of particle duality is noteworthy, in financial mathematics, etc.

The aim of the special subject on soliton is to bring together under one umbrella a wide variety of work on Solitons. Further we have reached the takeoff point where the practical applications of Solitons will now start to come. For example in the medical field Alzheimer’s disease has been found to be due to collective avalanche of neurons. Solitons have been noticed as waves in the sea and waves via oscillators in the air and have vehicles. They may occur in proteins and DNA and are related to the low-frequency collective motion in proteins and DNA. A recently enhanced model in neuroscience proposes that signals, in the form of density waves, are conducted within neurons in the form of solitons. Atomic nuclei may exhibit solitonic behavior. Here the whole nuclear wave function is predicted to exist as a soliton under certain conditions of temperature and energy. The bound state of two solitons is known as a bion or in systems where the bound state periodically oscillates. In field theory bion usually refers to the solution of the Born–Infeld model.

Actually the solitons will have two components: Theory and Applications. The common theme must be Solitons. Its applications fields are broadly and demand for modern ages and also create enough scopes to the researchers in mathematical sciences and different fields.

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Chaos Theory in Social Sciences and Literary Theory

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Chaos theory is a field of study in mathematics; however, it has applications in several disciplines, including sociology and other social sciences. In the social sciences, chaos theory is the study of complex non-linear systems of social complexity. It is not about disorder but rather about very complicated systems of order.

Chaos theory aims to find the general order of social systems and particularly social systems that are similar to each other. The assumption here is that the unpredictability in a system can be represented as overall behavior, which gives some amount of predictability, even when the system is unstable. Chaotic systems are not random systems. Chaotic systems have some kind of order, with an equation that determines overall behavior.

Some researchers in the field of social sciences (Kiel and Elliott 1996) even propose that the chaos theory offers a revolutionary new paradigm, away from the materialistic Utopia, and that social system should be maintained *at the edge of chaos,* between too much and too little authoritarian controls. This comment concerns politics rather than physics. The application of chaos models in the analysis of social phenomena is accompanied by some important scientific problems. First, whether observations of social phenomena are generated by nonlinear dynamics cannot be ascertained beyond considerable doubt, especially when these observations contain a measurement error that is there is a problem of external validity. Secondly, and more important, as a theory of irregular cyclical social behavior is lacking, inductive-statistical theory formation about such behavior, which is based on fitting a mathematical model of chaos to observations of social phenomena, is impossible unless additional information is used concerning the context and circumstances wherein the social phenomena occur; that is the internal validity of any theoretical explanation that is derived from only a fitted mathematical model (of chaos) cannot be assessed. So research into the suggestion derived from mathematical chaos theory that irregular cycles may be present in the development of social phenomena over time requires theory-formation about irregular cyclical social behavior on the basis of established theoretical insights and empirical evidence instead of fitting sophisticated mathematical models of chaos to observations of social phenomena.

As a literary theory (Yasser, KRA. 2007) chaos theory helps readers more deeply understand and appreciate the complex ideas behind some works of literature we might encounter. For example, Shakespeare's Hamlet, in many respects, perfectly illustrates many of the core principles of chaos theory. Hamlet himself, in fact, seems to possess a particular awareness of the chaotic nature of human existence. Throughout the play, Hamlet constantly questions not only his own motives and action and their possible ramifications and effects, but also those of the various forces both those that are natural and apparently supernatural that are conspiring around him. Hamlet itself highlights, in miniature, the various seemingly unpredictable and chaotic forces that control reality. In a sense, the play itself makes use of a version of the famous butterfly effect that would be postulated more than three centuries later: The death of Hamlet's father results, ultimately, in the utter collapse of the entire kingdom of Norway and the death of nearly every major character in the play. The entire world in which Hamlet lives – his entire reality, in fact both external and internal – is depicted as being radically shifted by the death of a single human being. The events that are depicted and examined in the play then, illustrate the chaotic, complex, and ultimately unpredictable and seemingly random and determined forces upon which reality is structured. Reading Shakespeare's Hamlet with a firm knowledge of chaos theory to reveal a surprising measure of awareness on Shakespeare's that predates the scholarly exploration of chaos theory by nearly four centuries.

Actually Chaos theory is a new way of thinking about what we have. It gives us a new concept of measurements and scales. It looks at the universe in an entirely different way. Understanding chaos understands life as we know it. Because of chaos, it is realized that even simple systems may give rise to and, hence, be used as models for complex behavior. Chaos forms a bridge between different fields.

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মানব জীবনের পরতে পরতে গণিত, উদ্ভাবনের সূতিকাগার

প্রথমেই ধন্যবাদ জানাই গণিত বিভাগ, খুলনা প্রকৌশল ও প্রযুক্তি বিশ্ববিদ্যালয়কে যাদের ঐকান্তিক প্রচেষ্টায় Application of Mathematics in different fields শিরোনামে একটি workshop আয়োজন হতে চলেছে। একই সাথে এই আয়োজনকে সুন্দর রুপ দেওয়ার প্রত্যয় নিয়ে কিছু লেখা আহ্বান করা হয়েছে যা অত্যন্ত ভালো উদ্যোগ বলে প্রতীয়মান হবে এই আশা রাখি। পাশাপাশি এমন চিন্তা – ভাবনাকে আমি সাধুবাদ জানাই এবং এর সফলতা কামনা করি।

বিজ্ঞানের অন্যতম শাখা গণিত যা গাণিতিক পদ্ধতির সাথে সম্পর্কযুক্ত। মানুষের সহজাত স্বাভাবিক বৈশিষ্ট্য হল গণনা, পরিমাণ নির্ণয় এবং যুক্তি প্রয়োগ করে বিশ্বকে অনুধাবন করতে শেখা। আর এই অনুধাবনকে বিভিন্ন কর্মকাণ্ডের মাধ্যমে যুক্তিসংগত ব্যাখ্যা ও এর পরিধি নির্ণয়ে মানুষের কর্ম প্রচেষ্টার ফলই হল গণিত । গণিতের উন্নতির সাথে সাথে মানব সভ্যতার উন্নতি হয়েছে, আমাদের পরিপার্শ্বের যা কিছু নান্দনিক, সৌন্দর্যময় তার অন্তরালে রয়েছে গণিতের যুক্তিসমূহের সঠিক প্রয়োগ। যুগে যুগে গণিতের উন্নতি সাধনে অনেকের ভূমিকা পরিলক্ষিত হয়েছে। প্রমানস্বরূপ প্রফেসর ডঃ জামাল নজরুল ইসলাম, প্রফেসর সতেন্দ্যনাথ বসু, প্রফেসর মেঘনাথ সাহা, প্রফেসর আসগর কাদির, প্রফেসর আব্দুস সালাম সহ আরও অনেকের জীবনী অনুসন্ধান করতে গেলে আমরা দেখতে পাই প্রত্যেকে গণিত ব্যবহারের মাধ্যমে বিজ্ঞানের বিভিন্ন শাখায় গুরুত্বপূর্ণ অবদান রেখে গেছেন। প্রকৃত অর্থে গণিতকে বলা হয় বিজ্ঞানের মা এবং গণিত একমাত্র সার্বজনীন ভাষা যা শিক্ষা করা আমাদের সকলের জীবনে অনস্বীকার্য।

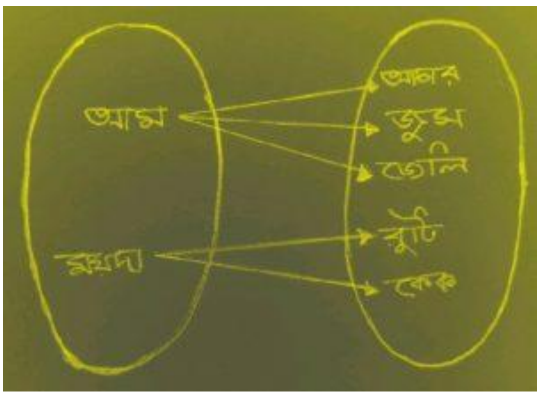
গণিতের ব্যবহার নেই এমন কোন কিছু ভাবা এক অর্থে অর্থহীন। গণিতের সংশ্লিষ্টতা মূলত তাত্ত্বিক পদার্থবিদ্যা ও ভৌতিক সমস্যার সাথে। একটু গভীরভাবে লক্ষ্য করলে দেখা যাবে যে পদার্থবিদ্যার মৌলিক নীতি গুলো ভৌতিক রাশির আচরণ নির্ধারক গাণিতিক সমীকরণের মাধ্যমে সূত্রবদ্ধ করা হয়। পাশাপাশি ব্যবহারিক দিক বিবেচনা করলে দেখা যায় আমাদের দেশে শিক্ষিত কিংবা অশিক্ষিত সকলেই কমবেশি গণিত জানে, জানতে হয় কারণ জীবনকে নিয়মমাফিক পরিচালনা করার জন্য কমবেশি গণিতের জ্ঞান প্রয়োজন। কয়েকটি উদাহরণের মাধ্যমে আমরা গণিতের প্রয়োজনীয়তাকে কিছুটা ব্যাখ্যা করতে পারিঃ

(i) গাণিতিক বাক্য যে শুধুমাত্র ভাব প্রকাশ করে তা নয়, এটি ‘দ্রুত’ ভাব প্রকাশে সাহায্য করে। আমাদের সেই ছোট বেলার কথা কি মনে পড়ে। স্কুল পর্যায়ের সেই লিচু ভাগাভাগির অংকগুলো? সেখানে বলা হতো, 30 টি লিচু তুমি দুজনের মাঝে এমনভাবে ভাগ করে দিবে যেন দ্বিতীয়জন প্রথমজনের চাইতে 5 টি লিচু বেশি পায়। আমরা ধরে নিতাম, প্রথমজন পাবে X টি লিচু এবং দ্বিতীয়জন পাবে Y টি লিচু, যেখানে $Y= X+5$. তাহলে তাদের মোট লিচুর সংখ্যা দাঁড়াবে $X+Y = 30$. আমরা একটু লক্ষ্য করলে দেখতে পাই, শেষের গাণিতিক সমীকরণ বা বাক্য উপরের কথাগুলোকে শুধু দুটি চলক ও একটি সংখ্যার মাধ্যমে প্রকাশ করে ফেলতে পারছে, যা স্পষ্টই দ্রুত ভাব প্রকাশে সাহায্য করছে।

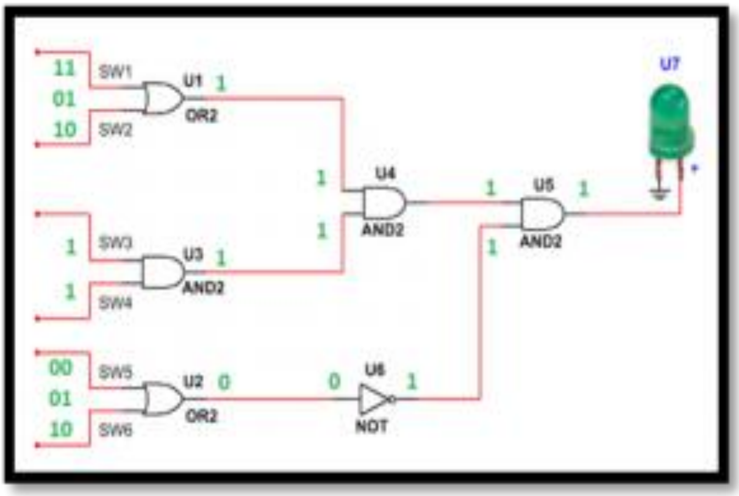
(ii) ধরুন, দেশের ভিতরে X এবং Y দুটি কোম্পানি। X কোম্পানি আম দিয়ে আমের আচার, আমের জুস, জেলি ইত্যাদি তৈরি করে থাকে। আবার ময়দা দিয়ে রুটি, কেকও তৈরি করে থাকে। কিন্তু সেই ময়দা X কোম্পানি Y কোম্পানি থেকে সংগ্রহ করে। এখানে আম আর ময়দা হলো কাঁচামাল, যেগুলোকে যন্ত্র দিয়ে প্রক্রিয়াজাত করে আচার, জুস, জেলি কিংবা রুটি, কেক উৎপাদন করা হচ্ছে। প্রক্রিয়া দুটিকে ফাংশনের আওতায় নিয়ে আসলে দাঁড়ায় –

X কোম্পানির যন্ত্র_১ = F (আম) = {আচার, জুস, জেলি}
Y কোম্পানির যন্ত্র_২ = G (গম) = {ময়দা}
X কোম্পানির যন্ত্র_৩ = H (ময়দা) = H(G(গম)) = {রুটি, কেক}
সুতরাং এখানে X কোম্পানির কাঁচামালের সেট = {আম, ময়দা}
এবং উৎপন্ন পণ্যের সেট = {আচার, জুস, জেলি, রুটি, কেক}

আমরা নিশ্চয় বুঝে গেছি যে, কাঁচামাল থেকে পণ্য উৎপাদন প্রক্রিয়াটি নিজের ভাষাতে বর্ণনা করতে গেলে খরচ করতে হবে অনেক শব্দ। কিন্তু গাণিতিক ভাষায় পুরো প্রক্রিয়াটি যেমন দ্রুত প্রকাশ করা সম্ভব, তার সাথে বর্ণনা করাও সহজ, যেখানে F, G, H অক্ষরগুলো যন্ত্রের মত কাজ করছে।



(iii) আবার ধরুন, আমরা বাড়িতে যে ফ্যান চালাই সুইচ টিপে, রিমোট ব্যবহারের মাধ্যমে এসি চালু করি, টেলিভিশনকে নিয়ন্ত্রণ করি ইত্যাদি ইত্যাদি ইত্যাদি। আসলে এগুলো হচ্ছে কী করে? আমরা কি কখনও ভেবে দেখি, কীভাবে এগুলো আমাদের নির্দেশনা মেনে চলছে এতো সুনিয়ন্ত্রিতভাবে? মূলত এর পেছনে কাজ করে গণিত, যাকে আমরা বলে থাকি বুলিয়ান গণিত। ইহার জন্য মাত্র দুটি সংখ্যা, 0 এবং 1 এর জন্য দায়ী। 0 মানে বন্ধ করা বা OFF এবং 1 মানে চালু করা বা ON.



প্রকৃত অর্থে চারপাশে তাকিয়ে দেখলে আমরা বিজ্ঞান সংশ্লিষ্ট এমন অনেক কিছুই পাবো যার ভিতরে গণিতের অবস্থান নিবিড়ভাবে জড়িয়ে রয়েছে। এই জন্য বর্তমান যুগকে বলা হয় বিজ্ঞানের যুগ, তথ্যপ্রযুক্তির যুগ। বিজ্ঞান ও তথ্যপ্রযুক্তির এত যে সাফল্য, এক কথায় বললে সেগুলো দাঁড়িয়ে আছে কতগুলো গাণিতিক সমীকরণ ও হিসাব নিকাশের উপর। তাইতো আমাদের জন্য গণিত অধ্যয়ন বিশেষ করে ফলিত গণিত অধ্যয়ন খুবেই অপরিহার্য হয়ে পরেছে। ফলিত গণিত এমনভাবে বিকশিত হয়েছে যার প্রভাব পড়েছে কম্পিউটার বিজ্ঞান (Computer Science), জীব গণিত, অপটিমাইজেশন তত্ত্ব (Optimization Theory), রসায়ন শাস্ত্র, পরিবেশ বিদ্যা, সঙ্কেত – লিপি বিজ্ঞান, সম্ভাবনা ও পরিসংখ্যান বিদ্যা, অর্থনীতি, অপারেশন রিসার্চ (Operation Research), প্রকৌশল সমস্যা, শিল্পবিদ্যা, প্রবাহ বল বিদ্যা (Fluid Dynamics), আপেক্ষিকতত্ত্ব (Theory of Relativity), সৃষ্টিতত্ত্ব (Cosmology), Meteorology, Aerodynamics ইত্যাদি শাখায়। অন্যদিকে বাস্তব জগতের নানাবিধ সমস্যায় যেমন জড় প্রক্রিয়াজাত (Material Processing), নকশা প্রনয়ন (Design), রোগ নির্ণয় ক্ষেত্রেও ফলিত গণিত শিক্ষা অপরিহার্য। একটি দেশে বিদ্যুৎ যেমন উন্নয়নের মাপকাঠি, ফলিত গণিত তেমনি দ্রুত উন্নতির সোপান। আর আমাদের এই সুবিধা পেতে হলে গণিত ছড়িয়ে দিতে হবে শিক্ষার সর্বস্তরে। এই বিষয়ে সকলের সহযোগিতা একান্তভাবে কাম্য।

পরিশেষে বলতে চাই, উপরের দুই – তিনটা উদাহরণের কথা লিখে Application of Mathematics in different fields এই বাক্যটিকে ব্যাখ্যা করে শেষ করা যাবে না। সময়ের সাথে সাথে নতুন নতুন উদ্ভাবন, প্রযুক্তির উৎকর্ষ সাধন, মানব জীবনে এর ব্যবহার সব কিছুতেই গণিতের প্রভাব আছে। আসলে মানব জীবনের পরতে পরতে গণিত ছিল comp সবসময়, হয়ত থাকবে আজীবন উদ্ভাবনের সূতিকাগার হয়ে। এই জন্য জগত বিখ্যাত বিজ্ঞানী গ্যালিলিওর ভাষায় বলতে হয়, *“আমাদের মহাবিশ্ব আসলে বিরাট এক গ্রন্থ। এর পরতে পরতে মিশে আছে দর্শন। গ্রন্থটা আমাদের চোখের সামনেই পড়ে আছে, কিন্তু একে বুঝতে হলে এর ভাষা আয়ত্ত করা চাই; ওই বর্ণগুলো চেনা চাই, যা দিয়ে লেখা হয়েছে এই বই। এটি লেখা হয়েছে গণিতের ভাষা দিয়ে, আর এর বর্ণ হল ত্রিভুজ, বৃত্ত ও অন্যান্য জ্যামিতিক চিত্র। এগুলো বাদ দিয়ে এর একটা শব্দও বুঝা সম্ভব নয়।”*

(মোঃ সুজন আলী, সহকারী অধ্যাপক)
গণিত বিভাগ, বঙ্গবন্ধু শেখ মুজিবুর রহমান বিজ্ঞান ও প্রযুক্তি বিশ্ববিদ্যালয়
গোপালগঞ্জ।

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