Lecture 10

Date: 02-03-2020
Table 6.1 Types of flow profiles in prismatic channels

<table>
<thead>
<tr>
<th>Channel Slope</th>
<th>Zone</th>
<th>Designation</th>
<th>Relation of $h$ to $h_n$ and $h_c$</th>
<th>Type of curve</th>
<th>Type of flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal $S_0 = 0$</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>H2</td>
<td>$h &gt; h_c$</td>
<td>Drawdown</td>
<td>Subcritical</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>H3</td>
<td>$h_c &gt; h$</td>
<td>Backwater</td>
<td>Supercritical</td>
</tr>
<tr>
<td>Mild $0 &lt; S_0 &lt; S_c$</td>
<td>1</td>
<td>M1</td>
<td>$h &gt; h_n &gt; h_c$</td>
<td>Backwater</td>
<td>Subcritical</td>
</tr>
<tr>
<td>$h_n &gt; h_c$</td>
<td>2</td>
<td>M2</td>
<td>$h_n &gt; h &gt; h_c$</td>
<td>Drawdown</td>
<td>Subcritical</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>M3</td>
<td>$h_n &gt; h_c &gt; h$</td>
<td>Backwater</td>
<td>Supercritical</td>
</tr>
<tr>
<td>Critical $S_0 = S_c &gt; 0$</td>
<td>1</td>
<td>C1</td>
<td>$h &gt; h_c = h$</td>
<td>Backwater</td>
<td>Subcritical</td>
</tr>
<tr>
<td>$h_n = h_c$</td>
<td>2</td>
<td>C2</td>
<td>$h_c = h = h_n$</td>
<td>Parallel to channel bottom</td>
<td>Uniform critical</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>C3</td>
<td>$h_c = h_n &gt; h$</td>
<td>Backwater</td>
<td>Supercritical</td>
</tr>
<tr>
<td>Steep $S_0 &gt; S_c &gt; 0$</td>
<td>1</td>
<td>S1</td>
<td>$h &gt; h_c &gt; h_n$</td>
<td>Backwater</td>
<td>Subcritical</td>
</tr>
<tr>
<td>$h_n &lt; h_c$</td>
<td>2</td>
<td>S2</td>
<td>$h_c &gt; h &gt; h_n$</td>
<td>Drawdown</td>
<td>Supercritical</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>S3</td>
<td>$h_c &gt; h_n &gt; h$</td>
<td>Backwater</td>
<td>Supercritical</td>
</tr>
<tr>
<td>Adverse $S_0 &lt; 0$</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>A2</td>
<td>$h &gt; h_c$</td>
<td>Drawdown</td>
<td>Subcritical</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>A3</td>
<td>$h_c &gt; h$</td>
<td>Backwater</td>
<td>Supercritical</td>
</tr>
</tbody>
</table>
General Procedure for Sketching Qualitative Flow Profiles

1. Draw the profile of the channel. Plot the critical depth line (CDL) and normal depth line (NDL), if any.

2. For the zone in which the profile lies, determine the relation of the depth $h$ to $h_c$ and $h_n$, if any. For example, for zone 1 of a mild slope channel, $h > h_n > h_c$.

3. Name the profile considering the channel slope and the zone in which it lies. For example, the name of the profile which lies in zone 2 of a steep slope channel is S2.

4. Determine the sign of $\frac{dh}{dx}$ from the signs of the numerator and denominator of the right hand side of Eq. (6.18). The numerator is positive if $h > h_n$ and negative if $h < h_n$. Similarly, the denominator is positive if $h > h_c$ and negative if $h < h_c$.

5. Determine whether the profile is a backwater curve or a drawdown curve. The profile is a backwater curve if $\frac{dh}{dx} > 0$ and a drawdown curve if $\frac{dh}{dx} < 0$.

6. Consider the conditions of the profile at its upstream and downstream ends. These conditions help to determine the actual shape of the profile, i.e. whether the profile is concave or convex.

7. Sketch the qualitative flow profile.

8. Determine whether the flow is sub critical or supercritical. Flow is sub critical if $h > h_c$ and supercritical if $h < h_c$. 
Flow Profiles in Mild Slope Channels ($S_0 > 0$ and $h_n > h_c$)

Using Eq. (6.18), the sign of $dh/dx$ in each zone can be determined as follows:

i) Zone 1:

$h > h_n > h_c$, $\frac{dh}{dx} = +\frac{dh}{dx} = +$, i.e. $\frac{dh}{dx} > 0$

ii) Zone 2:

$h_n > h > h_c$, $\frac{dh}{dx} = +\frac{dh}{dx} = -$, i.e. $\frac{dh}{dx} < 0$

iii) Zone 3:

$h_n > h_c > h$, $\frac{dh}{dx} = +\frac{dh}{dx} = +$, i.e. $\frac{dh}{dx} > 0$
The water surface profile in Zone 1, designated as M1, is a backwater curve and represents sub-critical flow.

At the upstream boundary \( (h \rightarrow h_n, \frac{dh}{dx} \rightarrow 0) \), the profile is tangential to the normal depth line and at the downstream boundary \( (h \rightarrow \infty, \frac{dh}{dx} \rightarrow S_0) \), the profile asymptotically approaches a horizontal line.

It may be noted that the water surface in an M1 profile falls in the downstream direction and approaches its horizontal asymptote from above.

The M1 profile occurs behind a dam, upstream of a weir or sluice gate in a mild slope channel, when a long mild slope channel ends in a reservoir to a depth greater than the normal depth and when a mild channel is followed by a milder channel.

The M1 profiles may be very long compared to other flow profiles. In rivers and canals, the M1 profiles may extend considerable distance before merging with the normal depth.

The M1 profile represents the most common flow profile and it is the most important flow profile from the practical point of view, since the slope of most rivers and canals is mild.
The M2 drawdown curve in Zone 2 is tangential to the normal depth line at its upstream boundary \( (h \rightarrow h_n, \frac{dh}{dx} \rightarrow 0) \) and normal to the critical depth line \( (h \rightarrow h_c, \frac{dh}{dx} \rightarrow \infty) \), indicating a hydraulic drop, at its downstream boundary.

This type of profile can occur at a free overfill, when a mild slope is followed by a steeper mild or critical or steep slope, when a mild slope channel ends in a reservoir to a depth less than the normal depth and at the upstream side of a sudden enlargement of a channel section.

The M3 backwater profile in Zone 3 starts theoretically from the channel bottom at its upstream end and terminates in a hydraulic jump at its upstream boundary \( (h \rightarrow h_c, \frac{dh}{dx} \rightarrow \infty) \).

The M3 profile occurs downstream of a sluice gate in a mild slope channel and when a supercritical flow enters a mild slope channel. The M3 profiles are relatively shorter than M1 and M2 profiles.
Flow profiles in Steep Slope Channels ($S_0 > 0$ and $h_n < h_c$)

- The S1 backwater profile begins with a hydraulic jump at the upstream boundary and tends to be horizontal at the downstream boundary.
- In the S1 profile the water surface rises in the downstream direction and approaches its horizontal asymptote from below.
- The S1 profile occurs behind a dam or upstream of a weir built in a steep channel and in a steep channel ending in reservoir to a depth more than the critical depth.

- The S2 drawdown curve starts from the critical depth line with a vertical slope at its upstream end and is tangential to the normal depth line at its downstream end.
- It is usually very short and acts like a transition between a hydraulic drop and uniform flow.
- This type of profile can occur downstream of an enlargement of a channel section and also downstream of a transition of slope from mild to steep or steep to steeper.

- The S3 backwater profile starts from the channel bottom and approaches the normal depth line tangentially.
- It may occur below a sluice gate on a steep slope or at a transition between steep slope and milder steep slope.
Fig. 6.3 Example of flow profiles
Flow Profiles in Critical Slope Channels \((S_0 > 0 \text{ and } h_n = h_c)\)

- In a critical slope channel the NDL and the CDL coincide since \(h_n = h_c\).
- Therefore, Zone 2 and the C2 profile which satisfy the condition \(h_n = h = h_c\) also coincide with the NDL and the CDL.
- The C2 profile thus represents uniform critical flow and may occur in a long prismatic critical slope channel.
- Since it represents uniform flow it is not considered as a profile of gradually varied flow.
- The C1 backwater profile in Zone 1 starts from the \(h_n = h_c\) line and tends to be horizontal downstream.
- The C3 backwater profile in Zone 3 starts from the channel bottom and meets the \(h_n = h_c\) line at its downstream end.
- Using the condition \(h_n = h_c\), Eq. \(\frac{dh}{dx} = S_0 \frac{1 - (h_n / h)^3}{1 - (h_c / h)^3}\) gives \(\frac{dh}{dx} = S_0\)
  - i.e. the C1 and C3 profiles in a wide channel are exactly horizontal.
- For channels which are not wide, generally \(M \approx N\).
- So, using the condition \(h_c = h_n\), Eq. (6.18) becomes \(\frac{dh}{dx} \approx S_0\)
  - indicating that the C1 and C3 profiles are approximately horizontal.
The C1 profile may occur upstream of a sluice gate on a critical slope or when a critical slope is followed by a mild or horizontal or adverse slope or it may connect a supercritical flow with a reservoir pool on a critical slope.

The C3 profile may occur downstream of a sluice gate in a critical slope channel or at the transition between steep and critical slopes.

The critical slope profiles are very rare.
Flow Profiles in Horizontal Channels \((S_0 = 0 \text{ and } h_n = \infty)\)

- For a horizontal slope \((S_0 = 0)\), Eq. \(K_n^2 = \frac{Q^2}{S_0} = \frac{Q^2}{0} = \infty\) gives \(K_n = \infty\) or \(K_n^2 = C_2 h_n^N \Rightarrow h_n = \infty\). Hence, \(h_n = \infty\), and, therefore, Zone 1 and an H1 profile satisfying the condition \(h > h_n > h_c\) are not physically possible.

- With \(S_0 = 0\),

  \[
  \frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2} = \frac{-(Q / K)^2}{1 - (h_c / h)^M}
  \]

  - so that the sign of \(dh/dx\) is obtained as follows:

    \[
    \begin{align*}
    i) & \text{ Zone 2: } h > h_c, \quad \frac{dh}{dx} = -+, \text{ i.e. } \frac{dh}{dx} < 0 \\
    ii) & \text{ Zone 3: } h < h_c, \quad \frac{dh}{dx} = -+, \text{ i.e. } \frac{dh}{dx} > 0
    \end{align*}
    \]

- The H2 drawdown profile has a horizontal asymptote at its upstream end \((h \to \infty)\) and ends in a hydraulic drop at its downstream end \((h \to h_c, \frac{dh}{dx} \to \infty)\).

- It may occur on a horizontal slope upstream of a free overfall.

- The H3 backwater profile, which is similar to the M3 profile, is obtained downstream of sluice gates and spillways on a horizontal slope. The horizontal slope profiles may be considered to be the limiting cases of the mild slope profiles when the channel becomes horizontal.
Flow Profiles in Adverse Slope Channels ($S_0 < 0$ and $h_n$ imaginary)

$$K_n^2 = \frac{Q^2}{S_0} = \frac{Q^2}{-} = -$$

$$K_n^2 = C_2 h_n^N$$

$$\Rightarrow h_n = i$$

$$\Rightarrow h_n = \text{imaginary}$$

$$h_n = \text{imaginary}$$

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2} = \frac{-|S_0 + S_f|}{1 - (h_c / h)^M}$$

hence, the sign of $\frac{dh}{dx}$ is obtained as follows:

i) Zone 2:

$$h > h_c, \frac{dh}{dx} = - = -, \text{ i.e. } \frac{dh}{dx} < 0$$

ii) Zone 3:

$$h < h_c, \frac{dh}{dx} = - = +, \text{ i.e. } \frac{dh}{dx} > 0$$

- The A2 and A3 profiles are similar to H2 and H3 profiles and are very rare.
- Only short lengths of adverse slope profiles may be expected to occur in practice.
Fig. 6.3 Example of flow profiles
It is evident from Table 6.1 and Figs. 6.2 and 6.3 that the profiles in Zone 1 (i.e. M1, S1 and C1) and Zone 3 (i.e. M3, S3, C3, H3 and A3) are backwater curves and those in Zone 2 (i.e. M2, S2, H2 and A2) are drawdown curves.

All the profiles in Zones 1 and 2, excepting the S2 profile, represent sub critical flow and those in Zone 3 and the S2 profile represent supercritical flow.