Lecture 07

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If the edges have different roughness's, what happens to the roughness as a whole?
Satsunai River
Atsubetsu River (flood)
Divide into sections that are controlled by each junction,
The flow velocity $U$ is equal at each point in the cross section.
\[ R_1 = \frac{A_1}{P_1}, \quad R_2 = \frac{A_2}{P_2}, \quad R_3 = \frac{A_3}{P_3}, \ldots, R_k = \frac{A_k}{P_k}, \ldots, R_N = \frac{A_N}{P_N} \]

And Manning formula

\[ U = \frac{1}{n_1} \left( \frac{A_1}{P_1} \right)^{2/3} S_0^{1/2} = \frac{1}{n_2} \left( \frac{A_2}{P_2} \right)^{2/3} S_0^{1/2} = \frac{1}{n_3} \left( \frac{A_3}{P_3} \right)^{2/3} S_0^{1/2} = \ldots \frac{1}{n_k} \left( \frac{A_k}{P_k} \right)^{2/3} S_0^{1/2} = \ldots \frac{1}{n_N} \left( \frac{A_N}{P_N} \right)^{2/3} S_0^{1/2} = \frac{1}{n} \left( \frac{A}{P} \right)^{2/3} S_0^{1/2} \]

\[ \frac{A_1}{n_1^{3/2} P_1} = \frac{A_2}{n_2^{3/2} P_2} = \frac{A_3}{n_3^{3/2} P_3} = \ldots \frac{A_k}{n_k^{3/2} P_k} = \ldots \frac{A_N}{n_N^{3/2} P_N} = \frac{A}{n^{3/2} P} \]

\[ n = \left( \frac{\sum_{k=1}^{N} n_k^{3/2} P_k}{P} \right)^{2/3} \]

\[ \sum_{k=1}^{N} n_k^{3/2} P_k = n^{3/2} P \]
The roughness may vary along the perimeter of a channel as shown in Fig. 4.5 (a). Such a channel section is known as a channel section with composite roughness.

A good example of such a section is provided by a rectangular flume built with a wooden bottom and glass walls having different n-values for the bottom and the walls.

\[
n = \left( \frac{P_1 n_1^{3/2} + P_2 n_2^{3/2} + P_3 n_3^{3/2}}{P} \right)^{3/2}
\]
Example 4.15

The sides of a laboratory flume are made of glass \((n = 0.010)\) and the bottom is made of wood \((n = 0.015)\). The flume is rectangular with \(b = 1\) m and is laid on a slope of 0.001. Compute the discharge in the flume if \(h_n = 0.4\) m.

Solution

Designating the perimeter of the bottom as \(P_1\) and the combined perimeter of the two sides as \(P_2\), we have \(P_1 = 1\) m, \(P_2 = 2 \times 0.4 = 0.8\) m, \(P = P_1 + P_2 = 1.8\) m, \(n_1 = 0.015\), \(n_2 = 0.010\). Then

\[
A = 1 \times 0.4 = 0.4m^2, \quad R = A / P = 0.4 / 1.8 = 0.222m
\]

\[
n = \left( \frac{P_1 n_1^{3/2} + P_2 n_2^{3/2}}{P} \right)^{2/3} = \left( \frac{1 \times 0.015^{3/2} + 0.8 \times 0.010^{3/2}}{1.8} \right)^{2/3} = 0.013
\]

\[
Q = \frac{1}{n} AR^{2/3} S_0^{1/2} = \frac{1}{0.013} \times 0.4 \times 0.222^{2/3} \times 0.001^{1/2} = 0.36m^3 / s
\]
COMPOUND CROSS-SECTION

\[ Q_1 = \frac{1}{n_1} A_1 R_1^{2/3} S_0^{1/2} \]

\[ Q_2 = \frac{1}{n_2} A_2 R_2^{2/3} S_0^{1/2} \]

\[ Q_3 = \frac{1}{n_3} A_3 R_3^{2/3} S_0^{1/2} \]

\[ Q = Q_1 + Q_2 + Q_3 \]

\[ Q_1 = K_1 \sqrt{S_f}, \quad Q_2 = K_2 \sqrt{S_f}, \quad Q_3 = K_3 \sqrt{S_f} \]

\[ U = \frac{Q}{A} \]

\[ A = A_1 + A_2 + A_3 \]

\[ n = A R^{2/3} S_0^{1/2} / Q \]

\[ \alpha = \frac{\alpha_1 K_1^2 / A_1^2 + \alpha_2 K_2^2 / A_2^2 + \alpha_3 K_3^2 / A_3^2}{K^3 / A^2} \]

\[ \beta = \frac{\beta_1 K_1^2 / A_1 + \beta_2 K_2^2 / A_2 + \beta_3 K_3^2 / A_3}{K^3 / A} \]

(4.126)
Example 4.16

A channel consists of a main section and two side sections as shown in the following figure. Compute the total discharge and the mean velocity of flow for the entire section when \( n = 0.025 \) for the main section, \( n = 0.035 \) for the side sections and \( S_0 = 0.0001 \). Also, compute the numerical values of \( n \), \( \alpha \) and \( \beta \) for the entire section assuming that \( \alpha = 1.12 \) and \( \beta = 1.04 \) for the main and side sections.

**Solution**

The main and the two side (left and right) sections are separated by drawing vertical (dotted) lines as shown. The computed values of \( A, P, R, Q \) and \( K \) for the three sections and \( A, P, Q \) and \( K \) for the entire section are shown below.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Section} & A & P & R & Q & K \\
\hline
\text{Main} & 425.0 & 56.18 & 7.565 & 655.12 & 65512 \\
\text{Left} & 80.0 & 24.00 & 3.333 & 51.00 & 5100 \\
\text{Right} & 60.0 & 19.00 & 3.158 & 36.90 & 3690 \\
\hline
\Sigma & 565.0 & 99.18 & 743.02 & 74302 \\
\hline
\end{array}
\]

Hence, the total discharge, \( Q = 743.02 \) m\(^3\)/s.

The mean velocity \( U \), roughness coefficient \( n \), energy coefficient \( \alpha \) and momentum coefficient \( \beta \) for the entire section are obtained as follows.

\[
U = \frac{Q}{A} = \frac{743.02}{565.0} = 1.315 \text{ m/s}
\]

\[
n = \frac{AR^{2/3}S_0^{1/2}}{Q} = 565.0 \times \left(\frac{565.0}{99.18}\right)^{2/3} \times 0.0001^{1/2} / 743.02 = 0.024
\]

\[
\alpha = \frac{\alpha_1K_1^2/A_1^3 + \alpha_2K_2^2/A_2^3 + \alpha_3K_3^2/A_3^3}{K^2/A}
\]

\[
= \frac{1.12 \times 65512^2/425^2 + 1.12 \times 5100^2/80^2 + 1.12 \times 3690^2/60^2}{74302^2/565^2} = 1.236
\]

\[
\beta = \frac{\beta_1K_1^2/A_1 + \beta_2K_2^2/A_2 + \beta_3K_3^2/A_3}{K^2/A}
\]

\[
= \frac{1.04 \times 65512^2/425^2 + 1.04 \times 5100^2/80^2 + 1.04 \times 3690^2/60}{74302^2/565} = 1.90
\]
Pb.

A trapezoidal channel has a bottom width of 6.0 m, side slopes of 1.5H:1V, a depth of flow of 2.0 m, $n = 0.025$ and $S_0 = 0.0001$. Assuming that the flow is uniform, (i) compute $Q$, (ii) compute $C$, $f$, $\tau_0$ and $u^*$, and (iii) compute $k_s$, determine whether the channel boundary is smooth or rough and state if the Manning formula is applicable for computing flow in this channel. Assume that the velocity distribution is logarithmic.
Pb.

Trapezoid, \( b = 6 \text{m}, b = 1.5 \text{m}, h = 2 \text{m}, n = 0.025 \)

\( A = (b + b_h)h = (6 + 1.5 \times 2) \times 2 = 18 \text{ m}^2 \)

\( P = b + 2\sqrt{1 + \frac{s^2}{2}}h = 6 + 2\sqrt{1 + 1.5^2} \times 2 = 13.21 \text{m} \)

\( R = A/P = 18/13.21 = 1.36 \text{m} \)

i) \( Q = \frac{1}{2} A R^{1/2} S_o^{1/2} = \frac{1}{0.025} \times 18 \times 1.36^{1/2} \times 0.0001^{1/2} \)

\[ = 8.838 \text{ m}^3/\text{s} \]

ii) \( C = \frac{1}{n} R^{1/6} = \frac{1}{0.025} \times 1.36^{1/6} = 42.1 \text{ m}^{1/6} \)

\[ f = \frac{8g}{C^2} = \frac{8 \times 9.81}{42.1^2} = 0.044 \]

\[ \gamma_0 = \gamma RS_o = 8g RS_o = 1000 \times 9.81 \times 1.36 \times 0.0001 \]

\[ = 1.334 \text{ N/m}^2 \]

\[ u_y = \sqrt{\gamma RS_o} = \sqrt{9.81 \times 1.36 \times 0.0001} = 0.0365 \text{ m/s} \]

(iii) To compute \( k_s \), assume that the surface is hydraulically rough.

\[ \therefore \frac{U}{u_y} = 5.75 \log \frac{12.2R}{k_s} \]

\[ \therefore \frac{0.491}{0.0365} = 5.75 \log \frac{12.2 \times 1.36}{k_s} = 0.491 \text{ m/s} \]

\[ \therefore 2.339 = \log \frac{12.2 \times 1.36}{k_s} \]

\[ \therefore \frac{12.2 \times 1.36}{k_s} = 10^{2.339} = 218.518 \]

\[ \therefore k_s = \frac{12.2 \times 1.36}{218.518} = 0.0759 \text{ m} \]

\[ \therefore \frac{k_s u_y}{\nu} = \frac{0.0759 \times 0.0365}{10^{-6}} = 2391.4173 \]

Hence, the surface is hydraulically rough and the Manning formula is applicable.
Consider the following data for the Padma (Ganges) river at the Baruria station in Faridpur on the 2\(^{nd}\) July, 1989: \(A = 33,500 \text{ m}^2\), \(Q = 56,200 \text{ m}^3/\text{s}\) and \(B = 3820 \text{ m}\). Assuming that the flow is uniform, (i) compute \(n\), \(C\), \(f\), \(u^*\) and \(\tau_0\), and (ii) determine whether the channel boundary is smooth or rough taking the velocity distribution as logarithmic. Assume that the river is wide. Longitudinal slope of the river is 4 cm/km.
b) \( A = 33,350 \text{ m}^2, \quad Q = 56,200 \text{ m}^3/\text{s}, \quad B = 382 \text{ m}, \quad S_o = 4 \text{ cm/km} \)
\[ = 4 \times 10^{-2}/10^3 = 4 \times 10^{-5} = 0.000049 \]

Since the river is wide, \( R = D = h = \frac{A}{B} = \frac{33,350}{382} \)
\[ U = \frac{Q}{A} = \frac{56,200}{33,350} = 1.68 \text{ m/s} \]
\[ = 8.77 \text{ m} \]

i) \( Q = \frac{1}{n} AR^{1/2} S_o^{1/2} \)
\[ \therefore \quad n = \frac{A R^{1/2} S_o^{1/2}}{Q} = \frac{33,350 \times 8.77 \times 4^{1/2} \times 0.000049^{1/2}}{56,200} = 0.016 \]
\[ C = \frac{1}{n} R^{1/2} = \frac{1}{0.016} \times 8.77^{1/2} = 89.53 \text{ m}^2/\text{s} \]
\[ f = \frac{8g}{C^2} = \frac{8 \times 9.81}{89.53^2} = 0.010 \]
\[ u^* = \sqrt{9gS_o} = \sqrt{9 \times 9.81 \times 0.000049} = 0.059 \text{ m/s} \]
\[ \tau_0 = u^* S_o = 89.53 \times 0.000049 = 3.44 \text{ N/m}^2 \]

ii) \[ \frac{U}{u^*} = 5.75 \log \frac{12.2h}{k_s} \Rightarrow \frac{1.68}{0.059} = 5.75 \log \frac{12.2 \times 8.77}{k_s} \]
\[ \log \frac{12.2 \times 8.77}{k_s} = 4.952 \Rightarrow \frac{12.2 \times 8.77}{k_s} = 10^{4.952} = 89.536 \]
\[ \therefore \quad k_s = \frac{12.2 \times 8.77}{89.536} = 0.001195 \text{ m} \]
\[ \frac{k_s}{u^*} = \frac{0.001195 \times 0.059}{10^{-6}} = 70.5 > 70 \]

Hence, the surface is hydraulically rough.
Show that for a wide rough channel with logarithmic velocity distribution in the vertical, the Manning roughness coefficient $n$ may be expressed by

$$n = \frac{(r - 1)h^{1/6}}{5.57(r + 0.95)}$$

Where $r (= u_{0.2}/u_{0.8})$ is the ratio between the measured velocities at two-tenths and eight-tenths of depth.
For a rough channel, Eqs. (4.15) and (4.17) with \( k = 0.4 \) gives

\[
U_2 = \frac{U_x \ln \frac{Z}{Z_0}}{k} = 2.5 U_x \ln \frac{2}{0.033} \text{ks} = 5.75 U_x \log \frac{2}{k} \text{ks}
\]

\[
\therefore U_{0.2} = 5.75 U_x \log \frac{24h}{k} \quad (i) \quad \text{at } z = 0.2h
\]

\[
U_{0.8} = 5.75 U_x \log \frac{6h}{k} \quad (ii) \quad \text{at } z = 0.8h
\]

\[
\therefore \frac{U_{0.2}}{U_{0.8}} = \frac{\log 24h}{\log 6h} \quad \Rightarrow \quad r = \frac{\log 24 + \log h/ks}{\log 6 + \log h/ks} \quad (iii)
\]

\[
\Rightarrow \log \frac{h}{ks} = \frac{1.380 - 0.0338r}{r-1}
\]

Using \( R = h \) for a wide channel, Eq. (4.10) can be written as

\[
\frac{U}{U_x} = 5.75 \log \frac{12.2h}{ks} = 5.75 \log \frac{h}{ks} + 6.25 \quad (iv)
\]

Using (iii) in (iv) and simplifying, we get

\[
\frac{U}{U_x} = 5.75 \left( \frac{1.380 - 0.0338r}{r-1} - \frac{0.338 + \log h/ks}{0.338 + \log h/ks} \right) = 1.28 (r + 0.95)
\]

Combination of Eqs. (4.9), (4.7A), and (4.7F) will give

\[
S_t = S_0 \quad \text{and} \quad R = h \text{ gives}
\]

\[
\frac{U}{U_x} = \frac{C}{V^9} = \frac{R^{\frac{1}{6}}}{nV^g} = \frac{h^{\frac{1}{6}}}{3.13n} \quad (vi)
\]

Equating the right-hand sides of (v) and (vi), we get

\[
n = \frac{(r-1) h^{\frac{1}{6}}}{5.57 (r+0.95)} \quad \text{(Proved)}
\]
When the Manning formula is used, show the critical slope at a given normal depth \( h_n \) may be expressed by

\[
S_c = \frac{gn^2 D_n}{R_n^{4/3}}
\]
and that this slope for a wide channel is

\[
S_c = \frac{gn^2}{h_n^{1/3}} = \frac{n^2 g^{10/9}}{q^{2/9}}
\]

Where \( q \) is the discharge per unit width.
The critical slope is the slope for which the flow is both uniform and critical, i.e., $h_n = h_e$, $U_n = U_e$ and so on. It is obtained by using a uniform flow formula in which the critical flow condition is incorporated. Now, using the Manning formula, we can write

$$U_n = \frac{1}{n} R_n^{1/3} S_e^{1/2}$$

or, $$S_e = \frac{R_n^{4/3} U_n^2}{R_n^{4/3}} \quad (i)$$

But $U_n = U_e$ and $U_e$ is obtained using the critical condition as

$$U_e = \sqrt{g D_e} = \sqrt{g D_n}$$

or, $$U_n = U_e = g D_n \quad (ii)$$

Hence, combining (i) and (ii) we obtain

$$S_e = \frac{R_n^{4/3} g D_n}{R_n^{4/3}} = \frac{g n} {R_n^{4/3}}$$ (Proved)

Now, for a wide channel, $R = D = h$, i.e.,

$$S_e = \frac{g n}{h_n^{4/3}} = \frac{g n}{h_n^{4/3}}$$ (Proved)

But the critical depth in a wide channel is given by

$$h_c = \sqrt{\frac{q_n^2}{g}} = h_n = h_n^{1/3} = \left(\frac{q_n^2}{g}\right)^{1/3} = \frac{q_n^{2/3}}{g^{1/3}}$$

$$S_e = \frac{g n}{h_n^{4/3}} = g^{10/9} h_n^{11/9} - h_n^{9/10}$$