Lecture 06

Date: 17-02-2020
Computation of Normal Depth
Analytical Method

The normal depth is an important parameter in the analysis of open channel flow. It may be computed using the Manning or the Chezy formula when the channel section, the discharge Q, the bottom slope $S_0$ and the Manning’s $n$ or the Chezy’s $C$ are given. For wide and triangular channels, the following analytical (explicit) expressions for the normal depth can be easily obtained.

**a) Using the Manning formula**

i. Wide channel

$$h_n = \left( \frac{nq}{\sqrt{S_0}} \right)^{3/5}$$  \hspace{1cm} (4.52)

ii. Triangular channel

$$h_n = \frac{2^{1/4} (4 + s^2)^{1/8}}{s^{5/8}} \left( \frac{nQ}{\sqrt{S_0}} \right)^{3/8}$$  \hspace{1cm} (4.53)

**b) Using the Chezy formula**

i. Wide channel

$$h_n = \left( \frac{q}{C \sqrt{S_0}} \right)^{2/3}$$  \hspace{1cm} (4.54)

ii. Triangular channel

$$h_n = \frac{2^{1/5} (1 + s^2)^{4/10}}{s^{3/5}} \left( \frac{Q}{C \sqrt{S_0}} \right)^{2/5}$$  \hspace{1cm} (4.55)
Example 4.5

A wide channel with \( S_0 = 0.0025 \) carries a discharge of 3 m\(^2\)/s. Compute the normal depth and velocity (a) using the Manning formula when \( n = 0.020 \), and (b) using the Chezy formula when \( C = 45 \) m \(^{1/2}\)/s.

Solution

(a) Using the Manning formula

\[
h_n = \left( \frac{mq}{\sqrt{S_0}} \right)^{3/5} = \left( \frac{0.020 \times 3}{\sqrt{0.0025}} \right)^{3/5} = 1.12 m
\]

\[
U_n = \frac{q}{h_n} = \frac{3}{1.12} = 2.69 m/s
\]

(b) Using the Chezy formula

\[
h_n = \left( \frac{q}{C\sqrt{S_0}} \right)^{2/3} = \left( \frac{3}{45 \times \sqrt{0.0025}} \right)^{2/3} = 1.21 m
\]

\[
U_n = \frac{q}{h_n} = \frac{3}{1.21} = 2.48 m/s
\]
Example 4.7

For a triangular channel with side slopes of 2:1, a longitudinal slope of 0.0016 and n = 0.015, determine the normal depth if Q = 10 m$^3$/s.

**solution**

\[
AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.015 \times 10}{\sqrt{0.0016}} = 3.75 
\]

\[
\therefore \, sh^2 \left( \frac{sh}{2\sqrt{1+ s^2}} \right)^{2/3} = 3.75 
\]

\[
or, \, 2h_n^2 \left( \frac{2h_n}{2\sqrt{1+ 2^2}} \right)^{2/3} = 3.75 
\]

which gives \( h_n^{8/3} = 3.206 \)

\( \therefore \, h_n = 1.55 \, \text{m} \)

Then, \( A = sh^2 = 2 \times 1.55^2 = 4.79 \, m^2 \)

\( \therefore \, U_n = \frac{Q}{A_n} = \frac{10}{4.79} = 2.09 \, m / s \)
Trial - and - Error Approach

For other simple geometric channel sections, like the rectangular, trapezoidal, circular and parabolic sections, the computation of normal depth can be conveniently carried out by the trial-and-error solution of Eq. (4.39).
Example 4.8

For a rectangular channel with \( b = 6.0 \) m, \( n = 0.025 \) and \( S_0 = 0.0025 \), compute the normal depth and velocity if \( Q = 20 \text{ m}^3/\text{s} \).

Solution

\[
AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 20}{\sqrt{0.0025}} = 10.000
\]

Now assume several values of \( h \) and compute the section factor \( AR^{2/3} \) unit the computed value of \( AR^{2/3} \) is close to 10.000.

<table>
<thead>
<tr>
<th>( h ) (m)</th>
<th>( A ) (m(^2))</th>
<th>( P ) (m)</th>
<th>( R ) (m)</th>
<th>( AR^{2/3} )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>6.000</td>
<td>8.000</td>
<td>0.750</td>
<td>4.952</td>
<td>h too small</td>
</tr>
<tr>
<td>2.00</td>
<td>12.000</td>
<td>10.000</td>
<td>1.200</td>
<td>13.551</td>
<td>h too large</td>
</tr>
<tr>
<td>1.60</td>
<td>9.600</td>
<td>9.200</td>
<td>1.043</td>
<td>9.876</td>
<td></td>
</tr>
<tr>
<td>1.61</td>
<td>9.660</td>
<td>9.220</td>
<td>0.048</td>
<td>9.965</td>
<td>h closest</td>
</tr>
</tbody>
</table>

Hence, the normal depth, \( h_n = 1.61 \) m and the normal velocity

\[
U_n = \frac{Q}{A} = \frac{20}{9.66} = 2.07 \text{ m/s}
\]
Example 4.9

For a trapezoidal channel with \(b = 6 \text{ m}, \ s = 2, \ n = 0.025\) and \(S_0 = 0.001\), compute the normal depth and velocity if \(Q = 14 \text{ m}^3/\text{s}\).

Solution

\[
AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 14}{\sqrt{0.001}} = 11.068
\]

<table>
<thead>
<tr>
<th>(h(\text{m}))</th>
<th>(A(\text{m}^2))</th>
<th>(P(\text{m}))</th>
<th>(R(\text{m}))</th>
<th>(AR^{2/3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>8.000</td>
<td>10.472</td>
<td>0.764</td>
<td>6.684</td>
</tr>
<tr>
<td>2.00</td>
<td>20.000</td>
<td>14.944</td>
<td>1.338</td>
<td>24.288</td>
</tr>
<tr>
<td>1.30</td>
<td>11.180</td>
<td>11.814</td>
<td>0.946</td>
<td>10.776</td>
</tr>
<tr>
<td>1.31</td>
<td>11.292</td>
<td>11.858</td>
<td>0.952</td>
<td>10.929</td>
</tr>
<tr>
<td>1.32</td>
<td>11.405</td>
<td>11.903</td>
<td>0.958</td>
<td>11.084</td>
</tr>
</tbody>
</table>

Hence, the normal depth, \(h_n = 1.32 \text{ m}\) and the normal velocity

\[
U_n = \frac{Q}{A} = \frac{14}{11.405} = 1.23 \text{ m/s}
\]
Example 4.9

Compute the normal depth and velocity in a parabolic channel with \( Q = 20 \, m^3/s \), \( n = 0.025 \) and \( S_0 = 0.0025 \) when the profile of the channel is given by \( y^2 = 4z \).

Solution

\[
AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 20}{\sqrt{0.0025}} = 10.000
\]

For a parabolic channel, the top width \( B = 2y \) and \( h = z \). Hence, assume values of \( h \), compute \( y \) using the equation \( y^2 = 4z \) taking \( z = h \) and then compute \( B, A, P, R \) and \( AR^{2/3} \) as shown below.

<table>
<thead>
<tr>
<th>h (m)</th>
<th>y (m)</th>
<th>B (m)</th>
<th>A (m^2)</th>
<th>P (m)</th>
<th>R (m)</th>
<th>( AR^{2/3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>2.000</td>
<td>4.000</td>
<td>2.667</td>
<td>4.591</td>
<td>0.581</td>
<td>1.856</td>
</tr>
<tr>
<td>2.00</td>
<td>2.828</td>
<td>5.657</td>
<td>7.542</td>
<td>7.192</td>
<td>1.049</td>
<td>7.785</td>
</tr>
<tr>
<td>3.00</td>
<td>3.464</td>
<td>6.928</td>
<td>13.856</td>
<td>9.562</td>
<td>1.449</td>
<td>17.743</td>
</tr>
<tr>
<td>2.27</td>
<td>3.013</td>
<td>6.027</td>
<td>9.120</td>
<td>7.846</td>
<td>1.162</td>
<td>10.008</td>
</tr>
</tbody>
</table>

Hence, the normal depth, \( h_n = 2.26 \, m \) and the normal velocity

\[
U_n = \frac{Q}{A} = \frac{20}{9.060} = 2.21 m / s
\]
Example 4.10

A circular channel 2 m in diameter is laid on a slope of 0.001 and carries a discharge of $4 \text{ m}^3/\text{s}$. Compute the normal depth and velocity when $n = 0.013$.

Solution

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.013 \times 4}{\sqrt{0.001}} = 1.644$$

<table>
<thead>
<tr>
<th>$\omega$ (rad)</th>
<th>A (m$^2$)</th>
<th>P(m)</th>
<th>R(m)</th>
<th>AR$^{2/3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.079</td>
<td>1</td>
<td>0.079</td>
<td>0.015</td>
</tr>
<tr>
<td>2</td>
<td>0.545</td>
<td>2</td>
<td>0.273</td>
<td>0.229</td>
</tr>
<tr>
<td>3</td>
<td>1.429</td>
<td>3</td>
<td>0.476</td>
<td>0.610</td>
</tr>
<tr>
<td>4</td>
<td>2.378</td>
<td>4</td>
<td>0.595</td>
<td>1.682</td>
</tr>
<tr>
<td>3.94</td>
<td>2.328</td>
<td>3.94</td>
<td>0.591</td>
<td>1.639</td>
</tr>
<tr>
<td>3.95</td>
<td>2.337</td>
<td>3.95</td>
<td>0.592</td>
<td>1.647</td>
</tr>
</tbody>
</table>

Hence, $\omega_n = 3.95 \text{ rad}$ and the normal depth

$$h_n = \frac{d_0}{2} \left(1 - \cos \frac{\omega}{2}\right) = \frac{2}{2} \left(1 - \cos \frac{3.95}{2}\right) = 1.39 \text{ m}$$

and, the normal velocity

$$U_n = \frac{Q}{A_n} = \frac{4}{2.337} = 1.71 \text{ m/s}$$
Numerical Methods

The numerical methods, used for solving nonlinear algebraic equations involving a single variable, e.g. the method of bisection, the method of iteration, the method of false position, the secant method, the Newton-Raphson method etc., as stated in Art. 3.2, can be conveniently used to compute the normal depth for rectangular trapezoidal, circular and parabolic channel sections. The computation of normal depth using the bisection and the Newton-Raphson methods is considered here.
**Bisection method:** Suppose that we want to compute the normal depth in a channel for a given section, discharge Q, roughness coefficient n and bottom slope $S_0$. Then the function

$$f(h) = AR^{2/3} - A_n R_n^{2/3} = AR^{2/3} - nQ / \sqrt{S_0}$$

must be satisfied by some positive depth greater than say $h_{\text{min}}$ and less than say $h_{\text{max}}$. The normal depth is taken equal to $(h_{\text{min}} + h_{\text{max}})/2$ and $f(h)$ is determined. If $f(h)$ is positive then the root is less than $(h_{\text{min}} + h_{\text{max}})/2$ and the upper limit is taken as $(h_{\text{min}} + h_{\text{max}})/2$. On the other hand, if $f(h)$ is negative, then the lower limit is taken as $(h_{\text{min}} + h_{\text{max}})/2$. The procedure is repeated till the desired accuracy is attained.
Example 4.11

For a trapezoidal channel with $b = 6$ m, $s = 2$, $n = 0.025$ and $S_0 = 0.001$, compute the normal depth by the method of bisection if $Q = 14$ m$^3$/s.

Solution

$$A = (6 + 2h)h, \ P = 6 + 4.472h, \ R = A/P, \ A_n R_{n}^{2/3} = 11.068$$

$$f(h) = AR_{n}^{2/3} - A_n R_{n}^{2/3} = \frac{[(6+2h)h]^{5/3}}{(6+4.472h)^{2/3}} - 11.068$$

Initially the values of $h_{\text{min}}$ and $h_{\text{max}}$ are taken as 0 and 10m, respectively. The computation is carried out as follows.

<table>
<thead>
<tr>
<th>$h_{\text{min}}$</th>
<th>$h_{\text{max}}$</th>
<th>$h=(h_{\text{min}} + h_{\text{max}})/2$</th>
<th>$f(h)$</th>
<th>Root lies between</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>5</td>
<td>148.647</td>
<td>0 and 5</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>2.5</td>
<td>26.562</td>
<td>0 and 2.5</td>
</tr>
<tr>
<td>0</td>
<td>2.5</td>
<td>1.25</td>
<td>-1.041</td>
<td>1.25 and 2.5</td>
</tr>
<tr>
<td>1.25</td>
<td>2.50</td>
<td>1.875</td>
<td>10.381</td>
<td>1.25 and 1.875</td>
</tr>
<tr>
<td>1.25</td>
<td>1.875</td>
<td>1.5625</td>
<td>4.105</td>
<td>1.25 and 1.5625</td>
</tr>
<tr>
<td>1.25</td>
<td>1.5625</td>
<td>1.4063</td>
<td>1.394</td>
<td>1.25 and 1.4063</td>
</tr>
<tr>
<td>1.25</td>
<td>1.4063</td>
<td>1.3281</td>
<td>0.143</td>
<td>1.25 and 1.3281</td>
</tr>
<tr>
<td>1.25</td>
<td>1.3281</td>
<td>1.2891</td>
<td>-0.458</td>
<td>1.2891 and 1.3281</td>
</tr>
<tr>
<td>1.2891</td>
<td>1.3281</td>
<td>1.3086</td>
<td>-9.357</td>
<td>1.3086 and 1.3281</td>
</tr>
<tr>
<td>1.3086</td>
<td>1.3281</td>
<td>1.3184</td>
<td>-0.009</td>
<td>1.3184 and 1.3281</td>
</tr>
</tbody>
</table>

Hence, the normal depth, $h_n = 1.32$ m
Newton-Raphson method: Suppose we want to compute the normal depth in a channel for given section, Q, n and S₀. Obviously, when \( h = h_n \)

\[
AR^{2/3} - \frac{nQ}{\sqrt{S_0}} = 0 \quad \text{or}, \quad A^{5/3} - \frac{nQ}{\sqrt{S_0}} P^{2/3} = 0 \quad \text{(since } R = A/P)\]

If we now assume

\[
f(h) = A^{5/3} - \frac{nQ}{\sqrt{S_0}} P^{2/3}
\]

then \( f'(h) = \frac{5}{3} A^{2/3} \frac{dA}{dh} - \frac{nQ}{3 \sqrt{S_0}} \times \frac{2}{3} P^{-1/3} \frac{dP}{dh} = \frac{5}{3} A^{2/3} B - \frac{2nQ}{3 \sqrt{S_0}} P^{-1/3} \frac{dP}{dh} \quad \left( \therefore \frac{dA}{dh} = B \right) \quad (4.105)\)

For a given channel section, \( f(h) \) and \( f'(h) \) depend on the depth of flow only and hence can be easily evaluated.
Example 4.13
For a trapezoidal channel with b = 6 m, s = 2, n = 0.025 and S₀ = 0.001, compute the normal depth by the Newton-Raphson method if Q = 14 m³/s.

Solution
\[ A = (6 + 2h)h \quad P = 6 + 2\sqrt{5h} \quad \text{and} \quad B = 6 + 4h \]
\[ \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 14}{\sqrt{0.001}} = 11.068 \]
\[ f(h) = \left[ (6 + 2h)h \right]^{5/3} - 11.068(6 + 2\sqrt{5h})^{2/3} = 3.175\left[ (3 + h)h \right]^{5/3} - 17.569\left( 4 + \sqrt{5h} \right)^{2/3} \]
\[ f'(h) = 5.291(3 + 2h)[(3 + h)h]^{2/3} - 26.191(3 + \sqrt{5h})^{1/3} \]
The computation of normal depth is carried out as follows.

<table>
<thead>
<tr>
<th>h</th>
<th>f(h)</th>
<th>f'(h)</th>
<th>Δh = - f(h) / f'(h)</th>
<th>h = h + Δh</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>-20.979</td>
<td>51.579</td>
<td>0.407</td>
<td>1.407</td>
</tr>
<tr>
<td>1.407</td>
<td>7.493</td>
<td>89.555</td>
<td>-0.083</td>
<td>1.324</td>
</tr>
<tr>
<td>1.324</td>
<td>0.406</td>
<td>81.215</td>
<td>-0.005</td>
<td>1.319</td>
</tr>
<tr>
<td>1.319</td>
<td>0.003</td>
<td>79.094</td>
<td>-0.000</td>
<td>1.319</td>
</tr>
</tbody>
</table>

Hence, the normal depth, \( h_n = 1.32 \text{ m} \)
COMPUTATION OF NORMAL AND CRITICAL SLOPES

- The normal slope \((S_n)\) is the longitudinal slope of the channel that is required to maintain uniform flow in the channel. When the Manning formula is used

\[
S_n = \frac{n^2 U_n^2}{R_n^{4/3}} = \frac{n^2 Q_n^2}{A_n^2 R_n^{4/3}}
\]

- or, when the Chezy formula is used

\[
S_n = \frac{U_n^2}{C^2 R_n} = \frac{Q_n^2}{C^2 A_n^2 R_n}
\]

- Equations indicate that the normal slope depends on the channel section, the discharge, the depth and the channel roughness. Thus when the channel section, \(Q\), \(n\) or \(C\) and \(h_n\) are given, the normal slope can be obtained using Eqs.
COMPUTATION OF NORMAL AND CRITICAL SLOPES

• The critical slope \( (S_c) \) is the longitudinal slope of the channel for which the flow in the channel is both uniform and critical, i.e. uniform flow occurs in a critical state and \( S_n = S_c, \ U_n = U_c \) and \( h_n = h_c \). When the channel section, \( n \) or \( C \) and \( h \) or \( Q \) are given, the critical slope can be determined using the Manning formula as

\[
S_c = \frac{n^2 U^2}{R^{4/3}} = \frac{n^2 Q^2}{A^2 R^{4/3}}
\]

• or, using the Chezy formula as

\[
S_c = \frac{U^2}{C^2 R} = \frac{Q^2}{C^2 A^2 R}
\]

• When \( Q \) is given, the normal depth \( h_n \), which is also equal to the critical depth \( h_c \), is first computed using the critical condition and then the critical slope is computed using Eq. On the other hand, when, \( h_n (=h_c) \) is given, the mean velocity \( U \) or the discharge \( Q \) is first determined using the critical condition and then the critical slope is computed using Eqs.
Example 4.14

A rectangular channel has a bottom width of 6 m, $\alpha = 1.12$ and $n = 0.020$. (a) For $h_n = 1$ m and $Q = 11$ m$^3$/s, determine the normal slope. (b) Determine the critical slope for $Q = 11$ m$^3$/s. (c) Determine the critical slope for $h_n = 1$ m.

Solution

Rectangular channel, $b = 6$ m, $\alpha = 1.12$, $n = 0.025$

(a) $h_n = 1$ m, $Q = 11$ m$^3$/s

$$A = bh = 6 \times 1 = 6 m^2, \quad P = b + 2h = 8m, \quad R = A / P = 0.75m$$

$$\therefore S_n = \left( \frac{nQ}{AR^{2/3}} \right)^2 = \left( \frac{0.020 \times 11}{6 \times 0.75^{2/3}} \right)^2 = 0.0020$$

(b) $Q = 11$ m$^3$/s

$$h_c = \frac{3 \alpha Q^2}{gb^2} = \frac{3 \times 1.12 \times 11^2}{9.81 \times 6^2} = 0.73m$$

$$\therefore h_n = h_c = 0.73m$$

$$A = bh = 6 \times 0.73 = 4.36 m^2, \quad P = b + 2h = 6 + 2 \times 0.73 = 7.45m, \quad R = A / P = 0.58m$$

$$\therefore S_c = \left( \frac{nQ}{AR^{2/3}} \right)^2 = \left( \frac{0.020 \times 11}{4.36 \times 0.58^{2/3}} \right)^2 = 0.0053$$

(c) $h_c = h_n = 1$ m

$$A = bh = 6 \times 1 = 6 m^2, \quad P = b + 2h = 6 + 2 \times 1 = 8m, \quad R = A / P = 0.75m$$

$$\therefore U_n = U_c = \sqrt{gD_c / \alpha} = \sqrt{gh_c / \alpha} = \sqrt{9.81 \times 1 / 1.12} = 2.96 m / s$$

$$\therefore Q = AU_c = 6 \times 2.96 = 17.76m^3 / s$$

or, $Q = \sqrt{g / \alpha bh_c^{1.5}} = \sqrt{9.81 / 1.12 \times 6 \times 1^{1.5}} = 17.76 m^3 / s$

$$\therefore S_c = \left( \frac{nQ}{AR^{2/3}} \right)^2 = \left( \frac{0.020 \times 17.76}{6 \times 0.75^{2/3}} \right)^2 = 0.0051$$