Genetic Programming-Based Ordinary Kriging for Spatial Interpolation of Rainfall

Sajal Kumar Adhikary1; Nitin Muttil2; and Abdullah Gokhan Yilmaz3

Abstract: Rainfall data provide an essential input for most hydrologic analyses and designs for effective management of water resource systems. However, in practice, missing values often occur in rainfall data that can ultimately influence the results of hydrologic analysis and design. Conventionally, stochastic interpolation methods such as kriging are the most frequently used approach to estimate the missing rainfall values where the variogram model that represents spatial correlations among data points plays a vital role and significantly impacts the performance of the methods. In the past, the standard variogram models in ordinary kriging were replaced with the universal function approximator-based variogram models, such as artificial neural networks (ANN). In the current study, applicability of genetic programming (GP) to derive the variogram model and use of this GP-derived variogram model within ordinary kriging for spatial interpolation was investigated. Developed genetic programming-based ordinary kriging (GPOK) was then applied for estimating the missing rainfall data at a rain gauge station using the historical rainfall data from 19 rain gauge stations in the Middle Yarra River catchment of Victoria, Australia. The results indicated that the proposed GPOK method outperformed the traditional ordinary kriging as well as the ANN-based ordinary kriging method for spatial interpolation of rainfall. Moreover, the GP-derived variogram model is shown to have advantages over the standard and ANN-derived variogram models. Therefore, the GP-derived variogram model seems to be a potential alternative to variogram models applied in the past and the proposed GPOK method is recommended as a viable option for spatial interpolation. DOI: 10.1061/(ASCE)HE.1943-5584.0001300. © 2015 American Society of Civil Engineers.

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Introduction

Rainfall data are used as a fundamental input for most hydrologic analyses and designs for effective management of water resource systems. The accuracy of most hydrological applications mainly depends on how accurately the spatial variability and distribution of rainfall is assessed (Xu and Singh 1998; Balme et al. 2006). Accurate spatial distribution of rainfall can be estimated through a dense rain gauge network, which involves large costs (Goovaerts 2000). However, the rain gauge network is sparse in most cases and limited numbers of point rainfall measurements are available in space, which are inadequate to characterize the highly variable rainfall and its spatial distribution. In such situations, spatial interpolation is important to estimate rainfall data at unrecorded locations (i.e., missing rainfall data) on the basis of the observed rainfall data available at surrounding locations.

Deterministic, stochastic, and data-driven spatial interpolation methods have been employed in the past for estimation of missing rainfall data. Deterministic interpolation techniques that include traditional weighting methods such as inverse-distance weighting (IDW) have been extensively used for estimating missing rainfall data (e.g., ASCE 1996; Sullivan and Unwin 2003). Alternative deterministic interpolation methods such as regression (conventional least-squares) and time series analysis were also used for this purpose (Salas 1993; Daly et al. 1994). Although the deterministic interpolation methods have been improved over time, their limitations continue to exist. Readers are referred to Vieux (2001), Grayson and Gunter (2001), Teegavarapu and Chandramouli (2005), and Teegavarapu (2007) for these limitations.

Stochastic spatial interpolation is another commonly used technique. In particular, variance-dependent stochastic interpolation methods, belonging to the general family of kriging, have been widely applied in hydrological sciences for spatial interpolation of hydrologic variables (Vieux 2001). These methods are based on the principle of minimizing estimation variances at locations with no measurements available (Teegavarapu 2007). Kriging in various forms has been used to estimate missing rainfall data and to interpolate rainfall from point measurements available at surrounding stations (Ashraf et al. 1997; Dingman 2002).

Ordinary kriging remains one of the most preferred stochastic interpolation methods (Webster and Oliver 2007), which has been applied for estimating missing rainfall and areal rainfall distribution from point rainfall data. However, the main limitation arises from its dependency on the variogram model that defines the spatial variability of the data. Moreover, selection of an appropriate variogram model, finding the optimal parameters of the standard parametric variogram models (i.e., nugget, range, sill), and the computational burden involved are some difficulties associated with the traditional ordinary kriging.

More recently, data-driven models using evolutionary and biological principles, namely genetic algorithms (GA), artificial neural networks (ANN), adaptive neuro-fuzzy inference system (ANFIS), and support vector regression (SVR), were used with kriging for...
spatial interpolation of rainfall and missing rainfall estimation. For instance, Chang et al. (2005) applied fuzzy theory and GA for interpolation of rainfall to minimize the rainfall estimation error. Teegavarapu (2007) used ANN-based ordinary kriging (ANOK) through variogram modeling using ANN to obtain necessary kriging weights for estimating missing rainfall data. Teegavarapu et al. (2009) identified optimal functional forms using GA to estimate missing rainfall data. Kisi and Sanikhani (2015) examined ANN, ANFIS, and SVR methods for long-term monthly rainfall estimation. However, no analytical equations can be obtained from ANN and GA, and it is generally not easy to interpret the network weights obtained from the ANN-derived models (Muttil and Lee 2005).

Genetic programming (GP) (Koza 1992) is another evolutionary data-driven modeling technique, which can be used for function approximation. GP-inferred models have the advantages of generating simple expressions and thus offering some possible interpretations to the underlying process (Muttil and Lee 2005). The key advantage of GP is that it does not assume any a priori functional form of the solution. Several applications of GP in hydrology and water resources research have been reported, which include rainfall-runoff simulation and flow estimation (Savic et al. 1999; Whigham and Crapper 2001; Khu et al. 2001; Liong et al. 2002; Nourani et al. 2012; Rodriguez-Vázquez et al. 2012; Yilmaz and Muttill 2014); real time prediction of coastal algal blooms (Muttil and Lee 2005); Sivapragasam et al. 2010); friction factor, and Muttil 2014); ordinary kriging was referenced through a case study is then presented. The results and analysis are then summarized, and finally the conclusions are drawn.

Ordinary Kriging
Kriging in geostatistics refers to a family of generalized least-square regression methods (Isaaks and Srivastava 1989; Webster and Oliver 2007). It helps to estimate the unknown variable values at unobserved locations on the basis of the observed known values at surrounding locations. The general expression of ordinary kriging to estimate missing value of variable Z in space is given by

\[ Z_{\text{OK}}^m(x_0) = \sum_{i=1}^{n} w_{i,\text{OK}} Z(x_i) \]  

(1)

where \( Z_{\text{OK}}^m(x_0) \) refers to the estimated missing value of variable Z (rainfall in this study) at desired location \( x_0 \); \( w_{i,\text{OK}} \) is the kriging weights associated with the observation at location \( x_i \) with respect to \( x_0 \); and \( n \) indicates the number of observed data points.

Ordinary kriging is known as the best linear unbiased estimator (Teegavarapu 2007; Abo-Monasar and Al-Zahrani 2014). It is linear in the sense that its estimates are obtained from weighted linear combinations of observed data. And it is best in the sense that the error variance is minimized while performing the estimation of unknown value at target location. It is unbiased because it tries to have the expected value of the residual to be zero. The unbiased condition in the kriging estimates is ensured by enforcing a constraint on the kriging weights that is expressed by

\[ \sum_{i=1}^{n} w_{i,\text{OK}} = 1 \]  

(2)

The kriging weights \( w_{i,\text{OK}} \) mainly depend on the fitted variogram model \( \gamma_{\text{std}}(h) \), which is determined by solving an optimization problem having the \( (n + 1) \) number of simultaneous linear equations as given by

\[ \sum_{i=1}^{n} \gamma_{\text{std}}(h_{ij}) w_{i,\text{OK}} + \mu_Z^2 = \gamma_{\text{std}}(h_{ji}) \quad \text{for} \quad j = 1, \ldots, n \]

(3)

where \( \gamma_{\text{std}}(h_{ij}) \) and \( \gamma_{\text{std}}(h_{ji}) \) are the variogram values obtained from the fitted standard variogram (i.e., exponential, gaussian, spherical used in this study) models for the distance \( h_{ij} \) and \( h_{ji} \), respectively; \( h_{ij} \) is the distance between observed data points \( x_i \) and \( x_j \); \( h_{ji} \) is the distance between the observed data point \( x_j \) and the target data point \( x_0 \) (where estimation is desired); and \( \mu_Z^2 \) is the Lagrange multiplier. The kriging variance in ordinary kriging \( \sigma_{m,\text{OK}}^2(x_0) \) at target location \( x_0 \) is obtained by

\[ \sigma_{m,\text{OK}}^2(x_0) = \mu_Z^2 + \sum_{i=1}^{n} w_{i,\text{OK}} \gamma_{\text{std}}(h_{ji}) \]  

(4)

Computing Experimental Variogram
Kriging interpolation heavily depends on the appropriate selection of a variogram model that defines the spatial structure of the observed data. To accomplish this, an experimental variogram was computed at first on the basis of the observed data, which was modeled by mathematical functions to obtain the variogram model. The variogram model provides the analytical estimation of variogram values for any distance and allows a unique and stable solution for kriging weights (Webster and Oliver 2007). A variogram is a mathematical function of the distance and direction separating two data points used to quantify the spatial autocorrelation in region-alyzed variables (RV). An RV is a variable that can take data values according to its spatial location (Chebhi et al. 2011). Once a proper variogram model is selected, kriging can be employed for missing data estimation (rainfall in this study).

In order to compute an experimental variogram, a variogram cloud was initially generated from the observed data by using Eq. (5)
Fig. 1. (a) Typical experimental variogram for a finite set of discrete lags based on a typical variogram cloud; (b) a typical variogram model with associated parameters fitted to the experimental variogram

\[
\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} (Z(x_i + h) - Z(x_i))^2 \tag{5}
\]

where \(N(h)\) is the number of data pairs separated by a lag distance \(h\); \(x_i\) and \((x_i + h)\) represent observed data point locations separated by a distance \(h\); and \(Z(x_i)\) and \(Z(x_i + h)\) indicate data values \(Z\) observed at the corresponding locations \(x_i\) and \((x_i + h)\), respectively. A variogram cloud consists of the semivariance values \(\gamma(h)\) calculated on the basis of observations at any two data points using Eq. (5) plotted against distance values between them (Johnston et al. 2001; Robertson 2008).

The experimental variogram was obtained from the variogram cloud by subdividing it into several classes and computing an average for each class (Robertson 2008). A typical variogram cloud based on Eq. (5) and a typical experimental variogram from the variogram cloud with fitted model is shown in Fig. 1. The experimental variogram is, therefore, composed of variogram values \(\gamma(h)\) for a finite set of discrete lags (Wackernagel 2003). A lag is a line vector that separates any two data points (Johnston et al. 2001). A mathematical function was fitted to the experimental variogram to obtain an appropriate variogram model. Traditionally, the standard variogram functions were used to fit the experimental variogram, and in this study GP has been used in addition to the standard variogram models.

**Modeling Experimental Variogram by Standard Variogram Models**

Several standard parametric variogram models are possible depending on the shape of the variogram function including exponential, gaussian, spherical, circular, and linear (Johnston et al. 2001; Robertson 2008). Among them exponential, gaussian, and spherical models are the most widely used variogram models for kriging applications in hydrology. For modeling experimental variogram, these three standard variogram functions were fitted to the experimental variogram, which are given by

\[
\gamma_{\text{Exponential}}(h) = C_0 + C_1 \left[1 - \exp\left(-\frac{3h}{a}\right)\right] \tag{6}
\]

\[
\gamma_{\text{Gaussian}}(h) = C_0 + C_1 \left[1 - \exp\left(-\frac{3h^2}{a^2}\right)\right] \tag{7}
\]

\[
\gamma_{\text{Spherical}}(h) = C_0 + C_1 \left[1.5 \frac{h}{a} - 0.5 \frac{h^3}{a^3}\right] \tag{8}
\]

where \(C_0\), \((C_0 + C_1)\) and \(a\) represent nugget, sill, and range respectively, which are commonly called the variogram parameters. These variogram parameters define a variogram model (Fig. 1) and thus affect the kriging computation. The nugget is the semivariance value at zero lag distance and represents a discontinuity at the origin of the variogram model. Sill indicates the constant semivariance of the RV beyond the range. Range is a distance beyond which there is little or no autocorrelation among variables.

Varioam modeling by the standard variogram functions (i.e., exponential, gaussian, spherical) was performed using GS+ (Robertson 2008), a geostatistical analysis software package for environmental sciences, to obtain the best fitted parameters of the standard variogram models. The variogram parameters of the corresponding standard variogram models were inferred based on the experimental variogram. The variogram parameters (nugget, sill, and range coefficients) were iteratively changed to obtain the best fitted model. The best model was selected on the basis of the root-mean-square error (RMSE), mean absolute error (MAE), and coefficient of correlation (CC) values. The model with the highest CC and the lowest RMSE and MAE values was chosen as the best variogram model, which was finally used in interpolation by the traditional ordinary kriging method.

**Cross-Validation Test**

Proper selection of a variogram model greatly affects the robustness of spatial interpolation by kriging (Grayson and Gunter 2001). The adequacy of the standard parametric variogram models is often tested by using a cross-validation scheme (Webster and Oliver 2007). The cross-validation is a simple leave-one-out validation process (Haddad et al. 2013) that involves eliminating the data values one after another from the observed data and then determining each data value on the basis of the remaining data values by kriging interpolation with the adopted variogram model. Cross-validation statistics provide important evidence of performance measures that demonstrate the adequacy of the adopted variogram model to yield accurate interpolation results by kriging (Adhikary et al. 2015). In the current study, mean standardized error (MS), root-mean-square error (RMSE), average kriging standard error (MKSE), and root-mean-square standardized error (RMSS) statistics (Johnston et al. 2001) were used to select the final form of standard variogram models. They are briefly described as follows:

- MS was used to test the unbiasedness condition of the variogram model, which should be close to zero for the unbiased estimates.
- RMSE was used to test whether the prediction was close to the observed data, which should be zero to make the predicted data exactly same as the observed data.)
• MKSE, RMSE, and RMSS were used to test whether the prediction variability is assessed correctly, in which the closer values of MKSE and RMSE indicate correct estimation of the prediction variability. Additionally, RMSS value closer to 1 demonstrates that the estimation variance is consistent and the variability of the prediction is correctly assessed.

**Genetic Programming-Based Ordinary Kriging**

In the current study, genetic programming-based ordinary kriging (GPOK) method used the GP-derived variogram model instead of using the standard variogram models (i.e., exponential, gaussian, spherical) used in traditional ordinary kriging. The basic conceptual difference between the traditional ordinary kriging and the proposed GPOK methods is shown in Fig. 2. As can be seen in this figure, the GP-derived variogram model was combined with ordinary kriging by replacing the standard variogram models to develop the proposed variant of kriging, which is referred to as the GPOK method in this study. The main advantage of the proposed GPOK method is that the GP-derived model used with this method does not require identifying the variogram parameters in advance, unlike the standard variogram models in the traditional ordinary kriging method. Thus, the GPOK method can reduce the tedious trial and error job of identifying the optimal variogram parameters and can thereby decrease the computational complexity involved in the traditional ordinary kriging.

The general expression of GPOK to estimate missing value of variable \( Z \) in space is given by

\[
Z_{\text{GPOK}}^m(x_0) = \sum_{i=1}^{n} w_i^{\text{GPOK}} Z(x_i) \quad \text{with} \quad \sum_{i=1}^{n} w_i^{\text{GPOK}} = 1 \tag{9}
\]

where \( Z_{\text{GPOK}}^m(x_0) \) refers to the estimated missing value of variable \( Z \) (rainfall in this study) at desired location \( x_0 \); \( w_i^{\text{GPOK}} \) is the kriging weights associated with the observation at location \( x_i \) with respect to \( x_0 \); and \( n \) indicates the number of observed data points.

The kriging weights \( w_i^{\text{GPOK}} \) depend on the GP-derived variogram model \( \gamma_{\text{GP}}(h) \), which can be determined by solving and the optimization problem having the \( (n+1) \) number of simultaneous linear equations as given by

\[
\sum_{i=1}^{n} \gamma_{\text{GP}}(h_{ij}) w_i^{\text{GPOK}} + \mu_Z^{\text{GPOK}} = \gamma_{\text{GP}}(h_0) \quad \text{for} \quad j = 1, \ldots, n \tag{10}
\]

\[
\sum_{i=1}^{n} w_i^{\text{GPOK}} = 1
\]

where \( \gamma_{\text{GP}}(h_{ij}) \) and \( \gamma_{\text{GP}}(h_0) \) are the variogram values that come from the GP-derived variogram model; and \( \mu_Z^{\text{GPOK}} \) is the Lagrangian multiplier. The kriging variance in GPOK \( \sigma_{m,\text{GPOK}}^2(x_0) \) at target location \( x_0 \) can be estimated by

\[
\sigma_{m,\text{GPOK}}^2(x_0) = \mu_Z^{\text{GPOK}} + \sum_{i=1}^{n} w_i^{\text{GPOK}} \gamma_{\text{GP}}(h_0) \tag{11}
\]

**Modeling Experimental Variogram by Genetic Programming**

In this study, GP was used as a universal function approximator to fit the experimental variogram in order to obtain the GP-derived variogram model that offers a potential alternative to the standard variogram models to be used for kriging interpolation. Mathematically, the GP-derived variogram model \( \gamma_{\text{GP}}(h) \) is given by

\[
\gamma_{\text{GP}}(h) = f(\text{Average lag distance between stations}, h) \tag{12}
\]

Variogram modeling by GP was performed using \( \text{GPKernel} \), a software package developed by DHI Water and Environment (Babovic and Keijzer 2000) to develop the GP-derived variogram model. The \( \text{GPKernel} \) is a command line-based tool for finding the underlying functions (in mathematical form) on data. The model with the lowest RMSE and MAE values and the highest CC value was selected as the best GP-derived variogram model, which was finally used for interpolation by the GPOK method.

**Genetic Programming**

Genetic programming (GP) is a relatively new automatic programming technique for evolving computer programs to solve problems (Koza 1992). In engineering applications, GP is frequently used as a universal function approximator to approximate and interpret the underlying structure of either a natural or experimental process (Muttil and Lee 2005). The main advantage of using GP over other universal function approximators (like ANN) is that it provides the underlying structure of the process in the form of a mathematical function (or equation), which can be scientifically interpreted to derive knowledge about the process to be modeled. Moreover, it does not assume any a priori functional form of the solution.

The basic search strategy behind GP (Koza 1992) is a genetic algorithm (GA) (Holland 1975; Goldberg 1989). GP differs from the traditional GA because GP operates on parse trees instead of bit strings to approximate the equation that best describes how the output are related to the input variables. A parse tree is formed by a terminal set (the variables involved in the problem) and a function set (the basic operators used to form the function). The algorithm creates an initial population of randomly generated parse trees, which are derived from the random combination of input variables, random numbers, and functions in the function set. The functions
can include arithmetic operators (e.g., +, −, *, /), mathematical functions (e.g., sin, cos, exp, log), logical/comparison functions (e.g., OR, AND), etc. The population of potential solutions is then subjected to an evolutionary process and the fitness (a measure that demonstrates how well the problem is solved) of the evolved programs are assessed. Individual programs that best fit to the data are selected from the initial population and are subjected to generate better programs through crossover and mutation. Crossover is the process of exchanging parts of the best programs with each other and mutation is process of changing programs randomly to create new programs. Copying parts of the best programs exactly into the next generation is called reproduction. This evolution process is repeated over successive generations and is driven towards finding symbolic expressions describing the data. For a comprehensive description of GP from water resources perspective, readers are referred to Babovic and Abbott (1997a), Babovic and Keijzer (2000), Khu et al. (2001), Liong et al. (2002), and Muttil and Lee (2005).

**Satisfying Positive Definiteness Condition**

According to Isaaks and Srivastava (1989), experimental variogram can be modeled by any new mathematical function (use of GP in this study) other than the available standard models (e.g., exponential, gaussian, spherical), and can be fitted to the experimental variogram. However, developing new variogram models by GP through fitting of the experimental variogram may not result in a unique and stable solution for the kriging weights, if the positive definiteness condition is not satisfied (Teegavarapu 2007). Positive definiteness condition is expressed by

\[
\text{Var} \left[ \sum_{i=1}^{n} w_{i} Z(x_{i}) \right] = w^{T} C w \geq 0 \quad (13)
\]

where \( w \) is the weight matrix; and \( C \) is the covariance matrix that includes covariance values between the observed points and those between the observed points and the location at which estimation is desired. The condition in Eq. (13) demonstrates that the use of a covariance function \( C \) for the computation of the kriging variance of a linear combination of \( (n + 1) \) random variables \( Z \) must be positive (Wackernagel 2003). If the condition in Eq. (13) is satisfied, the presence of one unique and stable solution for the kriging weights can be confirmed. Also, this indicates that the covariance matrix \( C \) satisfies the positive definiteness condition. Therefore, estimation of the kriging variance is important in the variogram modeling process because kriging heavily relies on its appropriate estimation. It is important that the standard variogram models always satisfy the positive definiteness condition (Wackernagel 2003) and hence passing only the cross-validation test is sufficient before using them in kriging. Conversely, newly developed variogram models (the GP-derived model in this study) should pass the cross-validation test and positive definiteness condition simultaneously before using them in kriging.

**Application of Genetic Programming-Based Ordinary Kriging**

The traditional ordinary kriging and the proposed GPOK methods were implemented in this study to estimate missing rainfall data at a base station (Seville, station no. 11 in Fig. 3). The base station refers to a station with missing data where the estimation is desired through spatial interpolation of the data available from the surrounding stations. In order to test the performance and robustness of the proposed GPOK method, it was assumed that the data were missing at the base station (although the data were available). Traditional ordinary kriging were implemented through ArcGIS 9.3.1 software and its geostatistical analyst extension (Johnston et al. 2001) to estimate missing data through spatial interpolation.

Fig. 3. Case study area (middle Yarra River catchment) showing the location of rain gauge stations (Seville, station no. 11 is the base station at which missing rainfall data are to be estimated)
at the base station. The proposed GPOK was implemented through a customized spreadsheet application developed in the Microsoft Excel platform for missing data estimation at the base station through spatial interpolation. The performance of the traditional ordinary kriging and the proposed GPOK methods were compared using the commonly used and widely recognized error measures. In this study, root mean square error (RMSE) and mean absolute error (MAE) based on the observed and estimated rainfall data at the base station were used.

### The Case Study Area and Data Used

In the current study, application of the proposed GPOK method and its robustness over traditional ordinary kriging method for missing data estimation was demonstrated through a case study area. The case study area includes the middle segment of Yarra River catchment with an area of 1,511 km² located in Victoria, Australia, which is shown in Fig. 3. The Yarra River catchment lies north and east of Melbourne covering an area of 4,044 km², where the Yarra River acts as the only lifeline (Melbourne Water 2013). The catchment is divided into three distinctive sub-catchments (Fig. 3), namely Upper Yarra, Middle Yarra, and Lower Yarra segments based on the different land use patterns (Barua et al. 2012). The Upper Yarra segment of the catchment consists of mainly forested and mountainous areas with minimum human settlement. The Middle Yarra segment (case study area) of the catchment is notable as the only part of the catchment with an extensive flood plain, which is mainly used for agricultural activities. The Lower Yarra segment of the catchment is mainly characterized by the urbanized and mountainous areas with minimum human settlement. The Yarra River catchment were used for the analysis. Location of these rain gauge stations which are currently being operated and maintained by the Bureau of Meteorology (BoM), Australia is shown in Fig. 3. The average annual rainfall based on the collected rainfall data from 1980 to 2012 varied between 508.1 and 1,913.9 mm with highest rainfall in the southern, eastern, and north-eastern parts, and lowest rainfall in the north-western part of the study area. The wettest month is September (rainfall amount equal to 112.5 mm) with the highest rainfall variation, and February is the driest month (rainfall amount equal to 56.4 mm) with the second highest rainfall variation. The statistics of daily and annual rainfall data at different rain gauge stations for the period from 1980 to 2012 are given in Table 1. The elevation in the study area varies from 25 m (lowest mainly in the central, north-western, and western parts) to 1,243 m (highest mainly in northern, north-eastern, and eastern parts) with a mean elevation of 621 m above mean sea level.

### Results

#### Check for Normal Distribution

The normal distribution of data is at the basis of the kriging-based geostatistical approach (Barca et al. 2008). Ordinary kriging leads to an optimum estimator when data values are normally distributed because kriging technique assumes that data come from a stationary stochastic process (Johnston et al. 2001). Exploratory data analysis was performed initially to detect the presence of spatial trend in the observed data and it was found that there was no spatial trend in the mean daily rainfall dataset given in Table 1. The widely recognized Kolmogorov-Smirnov (K-S) test, a simple and straightforward statistical test to check the normality condition of a data sample, was used to confirm the normality of observed rainfall data for a 5% significance level. Details of the K-S test can be found in McCuen (2003). Data were accepted as normally distributed (i.e., accept the null hypothesis) if the K-S test statistic value was less than the corresponding critical value for the 5% level of significance. In this study, the null hypothesis is defined as the condition that depicts the data follows normal distribution. It is evident from the K-S test that the null hypothesis cannot be rejected at the 5% level.

#### Table 1. Statistics of Observed Daily and Annual Rainfall Values at Different Stations

<table>
<thead>
<tr>
<th>Station numbera</th>
<th>BOM identifier</th>
<th>Station name</th>
<th>Distance from base stationb (km)</th>
<th>Daily rainfall (mm)</th>
<th>Annual rainfall (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
<td>Standard deviationc</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>86142</td>
<td>Toolangi (Mount St Leonard DPI)</td>
<td>25.621</td>
<td>3.653</td>
<td>8.526</td>
</tr>
<tr>
<td>2</td>
<td>86366</td>
<td>Fernshaw</td>
<td>23.353</td>
<td>3.196</td>
<td>7.079</td>
</tr>
<tr>
<td>3</td>
<td>86009</td>
<td>Black Spur</td>
<td>26.103</td>
<td>3.569</td>
<td>8.257</td>
</tr>
<tr>
<td>4</td>
<td>86070</td>
<td>Maroondah Weir (Melbourne Water)</td>
<td>18.833</td>
<td>2.894</td>
<td>6.569</td>
</tr>
<tr>
<td>5</td>
<td>86385</td>
<td>Healesville (Mount Yule)</td>
<td>17.013</td>
<td>2.453</td>
<td>5.614</td>
</tr>
<tr>
<td>6</td>
<td>86363</td>
<td>Tarrawarra</td>
<td>15.889</td>
<td>2.318</td>
<td>5.792</td>
</tr>
<tr>
<td>7</td>
<td>86364</td>
<td>Tarrawarra Monastery</td>
<td>16.470</td>
<td>2.138</td>
<td>5.074</td>
</tr>
<tr>
<td>8</td>
<td>86219</td>
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<td>3.504</td>
<td>7.682</td>
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<td>5.163</td>
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<td>86229</td>
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<td>2.659</td>
<td>5.814</td>
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<tr>
<td>11</td>
<td>86367</td>
<td>Seville</td>
<td>—</td>
<td>2.520</td>
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<tr>
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<td>Powlettown DNRE</td>
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<td>3.766</td>
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</tr>
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<td>14</td>
<td>86059</td>
<td>Kangaroo Ground</td>
<td>25.120</td>
<td>2.170</td>
<td>5.187</td>
</tr>
<tr>
<td>15</td>
<td>86066</td>
<td>Lilydale</td>
<td>14.678</td>
<td>2.352</td>
<td>7.403</td>
</tr>
<tr>
<td>16</td>
<td>86076</td>
<td>Montrose</td>
<td>11.113</td>
<td>3.188</td>
<td>7.403</td>
</tr>
<tr>
<td>17</td>
<td>86106</td>
<td>Silvan</td>
<td>5.614</td>
<td>3.116</td>
<td>6.800</td>
</tr>
<tr>
<td>18</td>
<td>86072</td>
<td>Monbulk (Spring Road)</td>
<td>10.751</td>
<td>3.452</td>
<td>7.490</td>
</tr>
<tr>
<td>19</td>
<td>86266</td>
<td>Ferny Creek</td>
<td>14.688</td>
<td>3.741</td>
<td>8.288</td>
</tr>
</tbody>
</table>

### Notes

- Station numbers are same as those in Fig. 1.
- Base station is the station (Seville, station no. 11) at which estimation is required.
- SD = Standard deviation of rainfall data.

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of significance because the computed K-S test statistic value (i.e., 0.14465) was less than the corresponding critical value (0.30143) for the mean daily rainfall data from 19 stations. Thus, it can be concluded that the data is normally distributed at 95% confidence level and can be directly used for subsequent variogram modeling and kriging interpolation.

**Variogram Modeling**

**Estimation of Experimental Variogram**

For variogram modeling, first an experimental variogram was computed based on the binning process (Wackernagel 2003; Oliver and Webster 2014) from the observed mean daily rainfall data provided in Table 1. Binning is a two-stage process in which the variogram cloud was generated initially using Eq. (5) to understand the spatial variability of the data and then pairs of points in the variogram cloud were grouped into a bin (as explained in Fig. 1). A bin is a classification of lags where all lags with similar distance and directions are categorized into the same class or bin. A total of 171 \(19 \times 171\) data pairs were obtained in the variogram cloud for all directions based on the mean daily rainfall data from 19 stations. The variogram cloud is shown in Fig. 4; however, it was difficult to visualize spatial autocorrelation because the data points were congested. It was also difficult to model the variogram cloud by appropriate variogram functions. For this reason, the variogram cloud (Fig. 4) was converted to an experimental variogram through the binning process. Hence, the variogram cloud was grouped into different bins to obtain the experimental variogram with finite number of discrete lags so that they could have a common distance and direction with respect to their distance from each other. The average lag distance and semivariance values for all the data pairs in each bin were plotted as a single point on the experimental variogram plot.

Initially, the directional experimental variogram in the \(x\) and \(y\) directions was computed from the variogram cloud. However, it was found to be noisy because of insufficient data points in these directions. Consequently, an isotropic experimental variogram was computed by neglecting the anisotropy effect on the variogram parameters. Isotropy is a property of a natural process or data where directional influence is considered insignificant and spatial dependence (autocorrelation) changes only with the distance between two locations (Johnston et al. 2001). The experimental variogram developed from the observed mean daily rainfall data for a sequence of fourteen bins is presented in Fig. 5. As shown in the figure, the experimental variogram reaches the sill for higher distance among stations that can be often equal to the variance of the data. The computed experimental variogram was then modeled by the standard variogram functions (exponential, gaussian, spherical models in this study) and GP, which are discussed in the following sub-sections.

**Standard Variogram Models**

In this study, three standard variogram models (exponential, gaussian, spherical) were fitted to the experimental variogram for traditional ordinary kriging. For the variogram fitting, model parameters were estimated by automatic fitting methods using GS+ (Robertson 2008). As a first guess for the sill parameter, the value of sample variance was adopted. The first guess for the range parameter was adjusted graphically. The parameters that resulted in the best fitting performance on the basis of three goodness-of-fit measures, namely RMSE, MAE, and CC, were identified as the optimal variogram parameters and the corresponding models were selected as the best fitted variogram model. The functional forms obtained for the standard variogram models are given by Eqs. (14)–(16) and Figs. 5(a–c) show the plots of these variogram models with the corresponding optimal parameters

\[
\text{Exponential model: } \gamma(h) = 0.001 + 0.460 \left[ 1 - \exp\left( -\frac{3h}{36.96} \right) \right] \\
\text{Gaussian model: } \gamma(h) = 0.035 + 0.344 \left[ 1 - \exp\left( -\frac{3h^2}{(16.14)^2} \right) \right] \\
\text{Spherical model: } \gamma(h) = \begin{cases} 
0.001 + 0.381 \left[ \frac{1.5h}{20.38} - 0.5h \left( \frac{20.38}{20.38} \right)^3 \right] & \text{for } h \leq 20.38 \\
0.382 & \text{for } h > 20.38 
\end{cases}
\]

where \(h\) is the distance in km.

These functional forms of the standard variogram models provide variogram values \(\gamma_{\text{est}}(h)\) to determine the kriging weights by Eq. (3) and kriging variance by Eq. (4) in traditional ordinary kriging depending on the target location at which the estimation is desired. Results related to the performance of the fitted models are given in Table 2. It is evident from Table 2 that the gaussian variogram model was the best fitted standard variogram model considering all the goodness-of-fit measures. The best fitted variogram model may not generate unique and stable solutions for the kriging weights if they fail to pass the cross-validation test (Webster and Oliver 2007). Therefore, the final form of the fitted variogram models was selected after satisfying all the cross-validation criteria before using for kriging interpolation. The cross-validation results for each of the variogram model were derived and the quality of prediction by ordinary kriging was assessed. The sill and range parameters of the variogram model were changed to obtain the best cross-validation results. This is referred to as trial and error method for parameter estimation. The cross-validation test statistics for the fitted standard models given in Table 2 indicate that all these models [Equations (14)–(16)] satisfy the cross-validation criteria and are thus appropriate for use in interpolation by ordinary kriging.

**Variogram Model by Genetic Programming**

For experimental variogram modeling using GP, simple function sets (+, −, *, /, x², exp) commonly available in the standard variogram functions as described by Eqs. (14)–(16) were adopted for the GP runs in order to generate simple GP-derived variogram models. Another objective was that the evolved GP models based on these
function sets could exhibit a similar structure of the standard variogram models. The function sets significantly impact the final form of the GP-derived model (Muttil and Lee 2005). Minimization of the root mean square error (RMSE) was used as the objective function for the evolution of best fitted model by GP. In the current study, the optimal functional form of the GP-derived variogram model was obtained using a population size of 500, crossover rate of 0.95, mutation rate of 0.05, and the number of generations as 500.

The functional form obtained for the variogram model by GP is given by Eq. (17).

\[
\gamma(h) = \begin{cases} 
0.023 + 0.344 \left[ 1 - \exp \left( -\frac{38^2}{16.14^2} \right) \right] & \text{for } h < 4.393 \\
0.4206139 \exp \left( \frac{4.438975}{(h)^{1.6043}} \right) & \text{for } h \geq 4.393
\end{cases}
\]  

(17)

where \( h \) is the distance in km.

The purpose of the functional form obtained by GP is to provide variogram values \( \gamma_{GP}(h) \) for determining the kriging weights by Eq. (10) and kriging variance by Eq. (11) in GPOK depending on the target location at which the estimation is desired. Fig. 5 (d) shows the plot of the GP-derived variogram model, which followed patterns similar to that of the standard variogram models. It can also be seen from the functional form expressed by Eq. (17) that the GP-derived variogram model consists of a simple structure similar to that of the standard variogram (i.e., exponential, Gaussian, and spherical) models [Eqs. (14)–(16)]. This indicates the advantage of GP over ANN for variogram modeling because of its ability to build models with a simple structure. Therefore, the GP-derived variogram model seems to be an alternative to the standard variogram models. It was also found that fitting of the experimental variogram using GP was very quick because unlike ANN, GP did not require a predefined mathematical form or architecture to generate the variogram model. Moreover, the GP-derived variogram model did not require identifying the variogram parameters in

\[
\begin{array}{c|c|c|c|c}
\text{Variogram model} & \text{Goodness-of-fit measures} & \text{Cross-validation statistics} \\
& \text{RMSE} & \text{MAE} & \text{CC} & \text{MS} & \text{RMSE} & \text{MKSE} & \text{RMSS} \\
\hline
\text{Standard variogram models} & & & & & & & \\
\text{Exponential} & 0.0268 & 0.0211 & 0.972 & -0.1006 & 0.4691 & 0.4500 & 1.0424 \\
\text{Gaussian} & 0.0227 & 0.0187 & 0.978 & -0.1100 & 0.5403 & 0.5500 & 0.9824 \\
\text{Spherical} & 0.0247 & 0.0211 & 0.974 & -0.1215 & 0.3922 & 0.3960 & 0.9905 \\
\hline
\text{GP-derived variogram model by GP} & & & & & & & \\
\text{GP-derived} & 0.0204 & 0.0155 & 0.982 & -0.0155 & 0.3582 & 0.3840 & 0.9328 \\
\end{array}
\]

Fig. 5. Plot of the experimental variogram: (a) with the fitted exponential variogram model; (b) with the fitted gaussian variogram model; (c) with the fitted spherical variogram model; (d) with the fitted GP-derived variogram model

Table 2. Performance Measures and Cross-Validation Statistics for Fitted Variogram Models Used in Kriging

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advantage, unlike standard variogram models in the traditional kriging method. As a result, the GPOK method was free from the trial and error process of estimating variogram parameters, and thus significant reduction of the computational complexity was achieved by this method. This is an important advantage of using GP for variogram modeling in the GPOK method. Therefore, the GPOK method is recommended as an alternative to the traditional ordinary kriging method.

According to Teegavarapu (2007), the nugget coefficient of the variogram model is an important parameter that should be determined accurately for reliable estimates in kriging. In this study, the nugget coefficient was obtained by extending the GP-derived model to touch the ordinate for zero distance in the abscissa. On the basis of the dataset configuration of the experimental variogram, a standard gaussian function was found best and hence used for connecting the starting point of the GP-derived model with the ordinate [Eq. (17) and Fig. 5(d)]. For this reason, the GP-derived model initially takes the shape of a gaussian variogram for the lag distance lower than 4.393 km. The nugget coefficient was found as 0.023 for the GP-derived model, which can be directly read from the plot given in Fig. 5(d). It is important to recall that the final form of the GP-derived variogram model should be chosen after performing the cross-validation test and checking the condition of positive definiteness based on the GP-derived model. The cross-validation test statistics obtained for the GP-derived variogram model are given in Table 2. It is evident from the table that the GP-derived variogram model satisfies all the cross-validation criteria. This initially confirms that the nugget coefficient of the GP-derived variogram model was correctly estimated.

In order to test the positive definiteness condition, the kriging variance by the GPOK method was calculated based on the GP-derived variogram model for all 19 rain gauge locations. The result is presented in Fig. 6, and indicates that the kriging variance for all the stations is positive. This confirms that the GP-derived model and its corresponding covariance function satisfy the condition of positive definiteness as described by Eq. (13). This conclusively proves that the nugget coefficient obtained by the GP-derived variogram model was correct. This is another important advantage of using GP over ANN for variogram modeling because unlike GP, ANN usually generates variogram model and corresponding covariance function with complex mathematical structure. Results of the positive definiteness test indicate that the GP-derived variogram model can provide unique and stable solutions for the kriging weights in interpolation by kriging.

The rain gauge network is frequently designed based on the variance reduction approach using kriging-based geostatistical approach (Tsintikidis et al. 2002; Barca et al. 2008; Chebbi et al. 2011; Adhikary et al. 2015). In the current study, the GP-derived variogram model was combined with ordinary kriging to achieve the proposed form of GPOK method. As shown in Fig. 6, the variance values obtained by the GPOK method are lower in the locations (stations 2–8) where the network density is comparatively higher (as seen in Fig. 3) than in other locations of the catchment. It also shows that station 5 [Healesville (Mount Yule)] exhibits a variance value close to zero. Therefore, station 5 is a redundant station at that location and hence can be either relocated to a high variance area or removed from the network. A similar conclusion was reported by Adhikary et al. (2015), where they used the standard variogram models in traditional ordinary kriging method for the design of rain gauge network. This provides an important insight and implication for the potential use of the proposed GPOK method for rain gauge network design in which the GP-derived variogram model replaces the standard variogram models.

### Spatial Interpolation of Rainfall by Traditional Ordinary Kriging, ANNOK, and GPOK Methods

In the current study, the traditional ordinary kriging, ANNOK, and the proposed GPOK methods were applied to estimate the missing rainfall data at the base station through spatial interpolation for one-third (4,054 days) of the total historical data (12,054 days), and the performance of both methods was compared. In case of traditional ordinary kriging method, the three standard variogram models, as given by Eqs. (14)–(16), were used to determine the kriging weights using Eq. (3) and thus estimate the missing rainfall data at the base station using Eq. (1) through spatial interpolation. In case of GPOK method, the GP-derived variogram model as expressed by Eq. (17) was used to determine the kriging weights using Eq. (10) and thus estimates the missing rainfall data at the base station using Eq. (9) through spatial interpolation. Moreover, the kriging variance was estimated based on Eq. (11) using the GPOK method at the base station (Seville, station no. 11), which was found as 0.175 mm² (Fig. 6). The variance was positive and hence the condition of positive definiteness as described by Eq. (13) was satisfied by the GPOK method.

In the ANNOK method, ANN-derived variogram model was used to calculate the kriging weights. In order to derive the variogram model by ANN, a feed forward 1-2-1 ANN architecture was used in this study, which consists of one hidden layer with two hidden layer neurons. A trial and error approach was used to identify the optimal number of neurons in the hidden layer. The 1-2-1 ANN architecture includes one input neuron (distance), two hidden layer neurons and one output neuron (semivariance). In the hidden and output layers, a sigmoidal activation function was used for modeling the transformation of values across the layers. ANN generally requires large data sets for its satisfactory training and testing. Because the experimental variogram consists of only 14 data points, the ANN model cannot be fitted to the experimental variogram. Unlike ANN, GP is capable of fitting comparatively smaller data sets to approximate the underlying functions. This is an important advantage of GP over ANN for variogram modelling. ANN models were developed based on 171 data points obtained in the variogram cloud for all directions based on the mean daily rainfall data from the 19 stations (Fig. 4). The ANN model was trained, cross-validated, and tested based on 121 data points (70% of data), 25 data points (15% of data), and 25 data points (15% of data), respectively. The functional form of the ANN-derived variogram model was obtained as an additive form of nonlinear and linear combination of weights and sigmoidal activation functions. However, the functional form of the ANN-derived variogram model was not as simple in structure (mathematical form) as the

![Fig. 6. Variance obtained for all stations by the GPOK method](image-url)
standard variogram functions. Details of variogram modeling by ANN and ANNOK method can be found in Teegavarapu (2007).

Results obtained for the performance of the aforementioned three methods are given in Table 3. It can be observed from the table that ordinary kriging with the gaussian variogram model yielded the best performance when compared to ordinary kriging with remaining two standard variogram models (exponential, spherical). It is also seen that the performance of the proposed GPOK method outperformed the traditional ordinary kriging with all the three standard variogram models and ANNOK methods for estimating the missing rainfall values at the base station. When comparing the performance of the GPOK method with the best traditional kriging (ordinary kriging with the gaussian variogram model) method, a reduction in error of 7.9% for RMSE and 18.5% for MAE was obtained with the GPOK method. Therefore, it can be concluded that use of the GPOK over traditional ordinary kriging method for missing data estimation results in a significant reduction of error, and thereby provides more accurate estimates of rainfall at the base station. However, a reduction in error with the GPOK method was found to be 11.7% for RMSE and 26.3% for MAE when comparing the performance of the GPOK method with the ANNOK method to estimate the missing rainfall. It is also shown in Table 3 that the ANNOK did not provide any improvement in the estimation of rainfall when compared to the best traditional ordinary kriging method (ordinary kriging with gaussian variogram model). However, the performance of ANNOK is similar to the remaining two traditional ordinary kriging methods (ordinary kriging with the exponential and spherical variogram models).

The ANN-based ordinary kriging presented in Teegavarapu (2007) also did not provide any improvement in the estimation of rainfall when compared with the best traditional ordinary kriging method (ordinary kriging with the spherical variogram model) in terms of RMSE and MAE values. Hence, the GPOK method can be regarded as a suitable alternative spatial interpolation method.

An attempt was made in this study to compare the GPOK-derived weights for all the stations and CC estimated based on observed data among stations. Also, a comparison was made between the GPOK-derived weights and the distance of any station from the base station (Seville, station no. 11). The results obtained for both cases are presented in Fig. 7(a). It is shown in Fig. 7(a) that approximately 80% of the total weight was assigned to the six stations located nearest the base station. Another plot of the GPOK-derived weights and CC as shown in Fig. 7(b) demonstrates that there was no reasonable relationship exists between the kriging weights and CC. Therefore, it can be concluded that correlation is not the best measure of assigning weights to a station. Similar conclusions were reported by Teegavarapu (2007), where he adopted ANN-based OK for spatial interpolation of rainfall data for missing data estimation at the base station.

Inspection of the layout of stations (Fig. 3) and GPOK-derived weights can provide interesting insights into the estimation process of the kriging weights. When comparing the weights for stations 9, 10, and 17 (which surround the base station 11), it is seen that higher weights were associated with stations 10 and 17 compared with station 9. This indicates that station 9 is not providing sufficient information than that provided by stations 10 and 17 in the interpolation. It is shown in Fig. 7(a) that even though stations 3, 4, 5, and 7 are located far away from the base station, the weights assigned by the GPOK method to those stations were higher than that assigned to station 9, which is located close to the base station. It is also shown in Fig. 7(a) that stations 2 and 6 are enclosed by stations 3, 4, 5, and 7, which were assigned with relatively higher weights. This is caused by the layout of stations 2–6 in the network. The positive weights of stations 3, 4, 5, and 7 are actually corrected to a small value by the negative weights of stations 2 and 6 on the basis of the station configuration. In this way, stations 3, 4, 5, and 7 have little effect on the base station, which is practically feasible.

The reason for getting negative weights for some stations is that there is no constraint in the ordinary kriging algorithm to enforce the kriging process to take positive values for the kriging weights (Szidarovszky et al. 1987). Hence, a methodology should be developed for computing the positive values of the kriging weights (Teegavarapu 2007). Recall that summation of all the weights must be equal to 1, an essential criterion used in kriging to avoid any systematic bias in estimating the kriging weights for both traditional ordinary kriging and GPOK methods. According to the principle

Table 3. Performance Results of Traditional Ordinary Kriging, ANNOK, and GPOK Methods for Spatial Interpolation of Rainfall

<table>
<thead>
<tr>
<th>Method</th>
<th>RMSE</th>
<th>Improvement in RMSE by GPOK (%)</th>
<th>MAE</th>
<th>Improvement in MAE by GPOK (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OK&lt;sup&gt;E&lt;/sup&gt;</td>
<td>1.722</td>
<td>11.6</td>
<td>0.583</td>
<td>23.5</td>
</tr>
<tr>
<td>OK&lt;sup&gt;G&lt;/sup&gt;</td>
<td>1.653</td>
<td>7.9</td>
<td>0.547</td>
<td>18.5</td>
</tr>
<tr>
<td>OK&lt;sup&gt;S&lt;/sup&gt;</td>
<td>1.734</td>
<td>12.2</td>
<td>0.603</td>
<td>26.1</td>
</tr>
<tr>
<td>ANNOK</td>
<td>1.723</td>
<td>11.7</td>
<td>0.605</td>
<td>26.3</td>
</tr>
<tr>
<td>GPOK</td>
<td>1.522</td>
<td>—</td>
<td>0.446</td>
<td>—</td>
</tr>
</tbody>
</table>

Note: OK<sup>E</sup> = ordinary kriging with the exponential variogram model; OK<sup>G</sup> = ordinary kriging with the gaussian variogram model; OK<sup>S</sup> = ordinary kriging with the ANN-derived variogram model; GPOK = ordinary kriging with the GP-derived variogram model.
of kriging, stations located close to the base station receive high weights, and stations located further away from the base station receive lower weights. However, if the sample search neighborhood is very large, and because the sum of the weights must equal to 1 for unbiased estimates, stations located furthest away from the base station may be assigned with the negative weights, in order for the nearby stations to maintain their higher weights, and for the sum of the weights to be equal to 1. The layout of stations (i.e., distance between stations in the rain gauge network) is therefore vital for assigning the weights to different stations. A similar conclusion was also reported by Deutsch (1996).

As can be seen from the performance measures in Table 3, the proposed GPOK method exhibits lower values of errors (RMSE and MAE) than the corresponding values obtained for all three traditional ordinary kriging methods. The maximum rainfall values estimated at the base station by the best selected traditional ordinary kriging (ordinary kriging with the gaussian variogram model) and GPOK methods were 115.8 and 107.7 mm, respectively, while the maximum rainfall actually observed was 99.0 mm. This indicates that the GPOK method yields better estimation of rainfall than the traditional ordinary kriging methods. The estimated and observed rainfall depths at the base station by the traditional ordinary kriging, ANNOK, and GPOK methods are shown in Fig. 8. It is evident from the figure that the estimated rainfall values show a good agreement with the observed rainfall values with less error in both methods.

Fig. 8. Plot of observed and estimated rainfall values: (a–c) traditional ordinary kriging methods; (d) ANN based ordinary kriging (ANNOK) method; (e) GP-based ordinary kriging (GPOK) method
Positive Kriging

Kriging, in its native state, does not ensure getting positive weights and thereby positive estimates. Negative weights can be obtained in ordinary kriging as a part of the solution for satisfying the requirement of unbiased constraints of kriging algorithm. It is the variogram that determines the magnitudes of negative weights based on the degree of continuity of the variable (Isaaks and Srivastava 1989). In the case of GPOK method, negative weights (when assigned to high rainfall values) may lead to negative estimates of desired rainfall values at the base station, which does not make physical sense. Szidarovszky et al. (1987) and Deutsch (1996) suggest that negative kriging weights should be corrected if it is obtained as a part of the solution. Fig. 7(a) indicates that six negative weights (for stations 1, 2, 6, 8, 16, and 18) are obtained by the GPOK method. Negative weights obtained in the kriging process can be eliminated using a technique called as positive kriging (Barnes and Johnson 1984). Szidarovszky et al. (1987) suggests that inclusion of an additional constraint for weights, \( w_i \geq 0 \) where \( i = 1, 2, 3, \ldots, n \) in the kriging process confirms the estimation of positive weights. In this study, a variant of positive kriging technique given by Teegavarapu (2007) was adopted to restrict the kriging weights to nonnegative values. The objective function was the difference between the observed and estimated rainfall values by the GPOK method (using the GPOK-derived weights) over a given time period. The optimization approach given by Teegavarapu (2007) based on mathematical programming (optimization) formulation can be expressed as

\[
\text{Minimize} \quad \sum_{j=1}^{n} \left[ \sum_{i=1}^{N} (w_iZ_i^j) - Z_m^j \right]^2 \quad (18)
\]

Subject to

\[
\sum_{i=1}^{N} w_i = 1 \quad (19)
\]

\[
w_i \geq 0 \quad (20)
\]

where \( Z_m^j \) is the observed rainfall value at the base station (where estimation is desired); \( Z_i^j \) is the observed rainfall values at individual stations; \( j \); \( N \) is the number of stations excluding the base station; and \( n \) is the number of days (i.e., the specific time period for which data used in individual stations, \( j \)). The objective function described by Eq. (18) minimizes the difference between the observed and kriging-based estimates of rainfall values over a period of \( n \) days. The constraint expressed by Eq. (19) will ensure the unbiased estimates, whereas the additional inequality constraint defined by Eq. (20) will evaluate the nonnegative kriging weights.

The optimization formulation [Eqs. (18)–(20)] was solved using Microsoft Excel solver with initial weights obtained by the GPOK. The solver uses a generalized reduced gradient (GRG) nonlinear optimization algorithm for the optimal solution. The optimal positive weights obtained from solution of the formulation were 0.000, 0.000, 0.000, 0.045, 0.053, 0.000, 0.106, 0.000, 0.074, 0.344, 0.033, 0.000, 0.064, 0.044, 0.000, 0.238, 0.000, and 0.000 for stations 1, 2, 3, \ldots, 10, and 12, 13, \ldots, 18, respectively. Approximately two-thirds (8,000 days) of the total historical data was used for estimation of weights using the optimization formulation whereas estimation was carried out for the remaining one-third (4,054 days) of the total historical data. The RMSE and MAE values for the estimation period using these weights are 1.560 and 0.495, respectively. The results showed that nine of the weights were zero, indicating rainfall values recorded at these nine stations had no effect on the rainfall estimates at the base station.

The final weights obtained from the solution of the formulation can be obtained only if the weights generated by the GPOK method are used as starting values in the Microsoft Excel solver. When the zero weights were restricted with a lower bound of 0.01, then the optimized weights obtained from the solution were 0.010, 0.010, 0.010, 0.010, 0.010, 0.010, 0.171, 0.561, 0.010, 0.010, 0.010, 0.010, 0.118, 0.010, and 0.010 for stations 1, 2, 3, \ldots, 10, and 12, 13, \ldots, 18, respectively. In this case, the RMSE and MAE values for the estimation period were 1.550 and 0.474, respectively. The kriging variance estimated using Eq. (11) at the base station based on the nonnegative weights obtained through the formulation previously described for both cases were 0.267 and 0.258 mm², respectively. The results indicated that these new sets of positive weights ensured the estimation of positive variance. However, the obtained variance was not same as the one originally provided by the GPOK method (0.175 mm²), indicating that the underlying condition of kriging relevant to minimum positive variance was not satisfied. Deutsch (1996) indicates that the kriging variance increases when the corrected nonnegative weights are applied for estimation. In order to address this issue, Deutsch (1996) suggests that the original kriging variance should be used along with the new set of positive weights in such case for subsequent application. The optimization formulation expressed in Eqs. (18)–(20) may ensure feasible (i.e., local) or global optimum solution. The solution is sensitive to initial values assigned to the weights and the historical data length (i.e., number of years) used for minimizing the objective function. The former aspect is attributable to limitations of GRG algorithm used for optimization and the later relates to effect of data length on the relative distribution of weights among the stations and on the value of objective function. Even though the variant of positive kriging used in this study confirms positive weights, the main limitation of mathematical programming formulations that often violate physical reality with mere imposition of bounds on variables cannot be avoided (Teegavarapu 2007).

Summary and Conclusions

In this study, a new method of spatial surface interpolation method was proposed, which is referred to as the genetic programming-based ordinary kriging (GPOK) method. In the proposed GPOK method, GP was used as a universal function approximator to obtain the GP-derived variogram model that was used as an alternative to the standard variogram models in traditional ordinary kriging. This study finally evaluates the efficacy and robustness of the newly proposed GPOK method over traditional ordinary kriging as well as ANN-based ordinary kriging (ANNOK) method through a case study demonstration aimed at estimating missing rainfall data values with the objective of minimizing the estimation error. The proposed GPOK method provided the best performance of the spatial surface interpolation when compared with the traditional ordinary kriging and the ANNOK method. Therefore, this study concludes that the fusion of geostatistical (ordinary kriging) and data-driven (GP) techniques is effective and has a high potential for spatial surface interpolation. However, one should be cautious before using the GP-derived variogram model because the GP-derived variogram model may not always result in a unique and stable solution for the kriging weights, which is an important prerequisite condition of ordinary kriging. It is therefore emphasized that in the case of the variogram models derived by GP, two
mandatory criteria, namely cross-validation and condition of positive definiteness, should be satisfied to obtain an authorized GP-derived variogram model to be used in kriging.

On the basis of the results obtained in this study, the following conclusions can be drawn:

- The function approximation capability of GP produces the best fitted GP-derived variogram model compared with the standard models.
- The GP-derived model does not require the variogram parameters to be identified in advance as usually necessary in case of the standard variogram models. Therefore, use of GP as a universal function approximator for modeling the experimental variogram eliminates the time consuming and tedious job of trial and error for determining the optimal variogram parameters as necessary with the standard variogram models.
- The GP-derived variogram model can be considered as an appropriate alternative to the available standard variogram models because GP has the capability to generate variogram models with similar mathematical structure as the standard variogram models.
- It is evident from the obtained results that the performance of the proposed GPOK method gives the best performance when compared with the traditional ordinary kriging as well as the ANNOK method to estimate the missing rainfall values.

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